

# Firing Restrictions and Economic Resilience: Protect and Survive?\*

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## Abstract

Firing restrictions are in use throughout the developed world but their role in the transmission of macroeconomic shocks into the real economy is mostly unstudied. We illustrate the theoretical role of these policies as amplifiers of macroeconomic shocks via labor-misallocation-induced output losses following an adverse shock. We use our model to derive an aggregation result which features a labor misallocation term and conduct a simulation exercise which demonstrates how misallocation can drive total factor productivity (TFP) down during recessions. We then perform a quasi-natural experiment which utilizes global credit supply shocks to study this amplifying role using a panel of 21 OECD economies. We show that strict firing restrictions are associated with a weaker initial response of the labor market, which is followed by a stronger and more persistent decline in real output as well as a slower return of real activity to pre-shock levels. The stronger output decline can be mostly explained by a stronger fall in aggregate TFP, which supports our theoretical predictions.

*JEL classifications:* E02, E24, E32, E60, J08

*Keywords:* Employment protection, Factor Misallocation, Credit Supply Shocks, Economic resilience, Business cycles, Local projections

*Online Appendix* available at <https://www.tomerifergane.com>.

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# 1 Introduction

How do firing restrictions affect the transmission of macroeconomic shocks? Employment protection legislation (EPL) is a widely used set of policy devices in developed economies and it plays an important institutional role in modern labor markets. Most of the policy debate regarding EPL is centered around two main issues: its effects on long-term macroeconomic performance on the one hand, and its significance for microeconomic outcomes in the labor market, on the other hand.<sup>1</sup> However, the use of such a policy device in times of economic adversity may alter the impact of macroeconomic shocks, influence their transmission mechanisms, and affect recovery. To fix ideas, throughout this paper, the term EPL will relate only to firing restrictions on regular workers and the two will be used interchangeably unless when relating to a particular restriction.

**What This Paper Does.** Our aim in this paper is to explore the potential link between EPL and economic resilience. We use the latter term to refer to an economy's ability to withstand macroeconomic shocks. To accomplish this goal, this paper unfolds in three parts.

First, we demonstrate the capacity of firing restrictions to affect misallocation during a business cycle using a search and matching model which incorporates termination costs and advance notice. Our model builds upon the work of [Lagos \(2006\)](#), but allows for an endogenous choice of capital and includes a novel treatment of termination notice within a search model. The model enables us to derive an aggregation result that illustrates the capacity of firing restrictions to generate a cyclical decline in total factor productivity (TFP) stemming from labor misallocation, in addition to the policies' steady-state effects.

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<sup>1</sup>The literature on EPL is vast and encompasses various fields, ranging from labor and political economics to macroeconomics; our focus in this paper is on the macroeconomic aspect of EPL and therefore we do not discuss much of the literature concerning EPL from other perspectives. For a comprehensive overview of this literature see [Skedinger \(2010\)](#) or chapter 10 in [Boeri and van Ours \(2013\)](#).

Our aggregation result may be useful for future empirical works aimed at providing a more detailed decomposition of TFP and applying misallocation adjustments so as to construct purified aggregate technology measures as well as isolate the effects of different policies.

Second, we conduct a quantitative exercise that attempts to gauge the relative importance of the misallocation channel for cyclical dynamics. We calibrate our model to match key moments in an economy which features significant firing restrictions, namely France. We propose a novel (to the best of our knowledge) calibration strategy and show that it performs well in terms of matching steady-state moments of the earnings distribution which are not targeted moments. The quantitative exercise suggests that the cyclical misallocation channel is present and of meaningful magnitude. We relate our model to the literature on aggregate fluctuation in search models, i.e. Shimer's puzzle and discuss its effects in our setup.

Last, we utilize global shifts in credit conditions to conduct a quasi-natural experiment capable of uncovering the effects and propagation of such shifts into output and labor markets of economies exhibiting different levels of firing restrictions. The central motivation for our empirical approach rests on the fact that the recent global financial crisis had a considerable effect on developed economies and that most of these economies vary substantially with respect to their labor market policies. We carry out this analysis by estimating state-dependent impulse response functions to the shock for measures of real activity and labor market activity. Our identification approach adapts the local projections method developed in [Jorda \(2005\)](#) to a panel setting, as in [Auerbach and Gorodnichenko \(2012\)](#).

The main results from our empirical analysis can be summarized as follows. Firing restrictions reduce the initial effect of the shock on the labor market, leading to a smaller and slower rise in unemployment, a smaller drop in employment, and to more stability

in terms of labor-force participation. However, from roughly the 1.5-year mark onwards, economies under strict firing restrictions experience a stronger and more persistent decline in real output. The drop in output is in the opposite direction to the effect on employment and too fast and sizable to be accounted for by a differential decline in capital stock, which is consistent with a drop in TFP taking place under the more restrictive regime. Such a drop is indeed evident in the data and is statistically significant. We further demonstrate that this sequence of differential responses in the labor market, real output, and TFP is statistically significant and robust to various choices of specifications and samples. These results are in line with those of the simulation exercise and suggest a TFP decline that is larger but falls within the same order of magnitude as our quantitative exercise implies. Interpreted through the lens of our theory, this amplification mechanism has its roots in EPL's contribution to increased misallocation of labor following an adverse shock.

Our results are of particular policy importance for the current COVID-19 crisis. Our theory indicates that countries in which firing restrictions are pervasive may experience a larger drop in aggregate TFP over the next few years. To ameliorate this adverse effect, our results provide support for a relaxation of these restrictions, at least temporarily, as a part of an economic exit strategy for the current crisis.

**Related Literature.** This paper is most closely related to the literature on labor market institutions and their interaction with macroeconomic shocks. The work of [Blanchard and Wolfers \(2000\)](#) describes how changes in European unemployment data can be explained by the interactions the institutional factors in the labor market with various shocks. In addition to the long-term changes in unemployment, institutional factors had been linked to macroeconomic volatilities (e.g., [Gnocchi et al. \(2015\)](#) and [Rumler and Scharler \(2011\)](#)). The interaction between EPL and the business cycle has also been studied in [Nunziata](#)

(2003) which demonstrates empirically and theoretically that strictness of EPL lowers the output elasticity of employment. Along this line, Duval and Vogel (2008) illustrate how strict EPL leads to more persistence in business cycle dynamics using output gap to identify cycles. The mechanism suggested by theory to explain this link between cyclical adjustment and EPL is that strict EPL should slow turnover dynamics and make the adjustment process to a shock longer as in Bentolila and Bertola (1990) and in Garibaldi (1998). The work of Messina and Vallanti (2007) provides support to this claim using firm-level data which indicates that strictness of EPL dampens the response of job destruction to the cycle, thus leading to less counter-cyclical job destruction.

This paper is also related to the literature which emerged after the Great Recession aimed at understanding the fashion in which different advanced economies have responded to what was generally considered as a global shock. Just following the Great Recession Ohanian (2010) examines the way in which Europe and the United States have experienced this shock using business cycle accounting. Ohanian's analysis points to the fact that in Europe the drop in the productivity deviation was more pronounced, while the United States had experienced little change in the productivity deviation but had experienced a sizable drop in the labor deviation relative to that which was present in Europe. Ohanian notes that this may be due to European firing restrictions which may lead to labor hoarding and lower measured productivity. This insight, which is revisited in Ohanian and Raffo (2012), is another motivation for this paper as it will demonstrate in detail how this may be the case and to what extent is this channel present. The relevance of labor market rigidities to the propagation of international business cycles is also discussed in the work of Perri and Quadrini (2018) which show that when the authors account for a variation in the adjustment costs of the labor input between the United States and the G6 countries, their model is able to provide a better match for the response patterns from the Great Recession.

**Contribution.** The contribution of this paper are twofold. First, the paper contributes to the empirical literature by conducting a comprehensive investigation of the link between firing restrictions and the transmission of credit supply shocks to several outcome measures, such as real output, private consumption, investment, capacity utilization, TFP, unemployment, employment to population ratio, and labor-force participation. Our identification strategy in this paper differs from the aforementioned works due to the use of an identified shock and higher data frequencies to estimate non-linear, state-dependent impulse response functions which allow observing the restrictions' effect on the shock's transmission channel rather than exploring these effects on moments or long-term trends.

Second, this paper contributes to the theoretical literature by creating a very rich model of firing restrictions. The conceptual contribution of the model is the aggregation result provided in Section 2, which is a generalization of the result of Lagos (2006) with a richer institutional setup. Using a quantitative version of our model we illustrate the amplification channel by which firing restrictions lead to an increased level of misallocation following an adverse shock, thus lowering TFP and consequently leading to a more stark drop in output. Although we cannot directly observe the theoretical channel in the data, the model provides predictions that are broadly consistent with our empirical findings and allows for a better understanding thereof. As such, we view the empirical contribution of this paper as the most significant one, and will continue to explore the theoretical misallocation channel in our future work.

**Outline.** The rest of the paper proceeds as follows. We begin in Section 2 by modeling the way by which firing restrictions affect aggregate productivity and affect the transmission of aggregate shocks. We proceed in Section 3 by conducting a quantitative calibration and simulation exercise by which we illustrate the potential amplification effect that results from increased labor misallocation following an adverse shock. We then move on

to our empirical analysis. We describe the data used with an emphasis on the measure for EPL in Section 4. In Section 5 we present our econometric method. In Section 6 we present and discuss our results. The final section concludes. We relegate some of the more technical elements of the model to Appendix A, the robustness of our empirical results is discussed at length in Online Appendix B, and a detailed description of the datasets used is given in Online Appendix C.

## 2 Theory - Firing Restrictions and Cyclical Behavior

In this section, we illustrate how firing restrictions can affect the transmission of an aggregate shock, using a simple, one-sector model. Firing restrictions in our model will consist of a firing cost and a period of termination notice.<sup>2</sup> During the notice period, the worker awaits termination and thus has no incentive to exert effort in production. The firm is bound by legal constraints to continue employing said worker under the same wage. Total separation costs from an employee are thus the sum of the cost of firing and wage paid for the duration of the notice period. In terms of aggregate production, this separation cost can be conceived as an adjustment cost associated with the aggregate labor input. The more costly the adjustment is, the less likely it is to occur, which means that the firm will be less inclined to separate from less productive workers. This incentive lowers aggregate productivity which is the average productivity of all matches.

This link between separation costs and productivity is presented in Lagos (2006) which shows that firing cost reduce aggregate steady-state productivity. His analysis builds on the framework of the textbook endogenous separation search and matching model found

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<sup>2</sup>This is not a normative paper so we do not model why does this regulation exist for a comprehensive treatment of this issue see Saint-Paul (2000). The key intuition is that in a frictional economy there are rents associated with employment and the median voter is likely to be an employed person trying to maintain or seek rents.

in [Pissarides \(2000\)](#) and links the reservation productivity level, the lowest productivity realization of a match that does not result in termination, and aggregate productivity. The lower the reservation level, the lower is aggregate TFP. Our model follows the two previous models closely but with the following noteworthy alterations.

First, we add to the model termination notice instead of just a firing cost. This extension, in itself, is not novel and had been implemented by [Garibaldi \(1998\)](#) and [Bentolila et al. \(2012\)](#). We use the same mechanics to allow for a delayed firing mechanism but, importantly, we endogenize the wage paid during the notice period.<sup>3</sup> Upon the choice of a matched pair to separate, the worker produces the minimum possible amount and is paid the last wage earned by her until a firing-permission arrives and induces payment of the firing cost by the firm and final separation of the pair. Second, since we are interested in business cycle dynamics, we add aggregate risk into the model. Third, since our empirical analysis will consist of an impulse response to a risk premium shock, we add a capital choice at the individual job level into the model and a reduced-form risk premium shock.

Our theoretical device abstracts from many potential channels of influence for firing restrictions. We abstract from the policies' potential impact on research and development expenditure as in [Saint-Paul \(2002\)](#), from the potential for distributional effects as in [Kahn \(2007\)](#), from nominal rigidities as in [Zanetti \(2011\)](#), and from their effect on long-term human capital accumulation as in [Gaetani and Doepke \(2016\)](#). The reason for this simplification is twofold. The first is analytical tractability and the second is that most of these elements have a bearing on long-term growth and market structure while our key interest is cyclical dynamics for shorter horizons. Hence, the merits of using a tractable search

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<sup>3</sup>[Garibaldi \(1998\)](#) assumes that the firm is able to extract the full rent from the employee, so there is no bargaining, and [Bentolila et al. \(2012\)](#) assume that the wage paid during notice is the same as the average wage in the economy. We allow the firm-worker pair to bargain during the regular employment period using standard Nash bargaining but, given the knowledge that regulation imposes upon the firm to continue paying the bargained wage to the worker until the end of the notice period.



and matching model as a theoretical device outweigh, in our eyes at least, its inherent limitations.

## 2.1 The Model

A firm in the model is an employer-employee pair which produces a single homogeneous good using capital,  $k$ , a common productivity factor,  $p$ , and an idiosyncratic component,  $x$ , which quantifies efficiency units of labor. Efficiency units of labor at the individual job level are drawn from a common primitive distribution with CDF  $G(x)$  and a compact support  $[x_{\min}, x_{\max}]$ . Each job may experience an idiosyncratic shock that arrives at rate  $\lambda$  which re-draws  $x$  from  $G(x)$ . The arrival of such a shock may trigger a separation choice. We assume that the match cannot separate immediately due to firing restrictions but that the separation decision results in the pair entering into a period of termination notice. The worker under notice receives her last wage until separation occurs. This worker produces with the minimum amount possible of efficiency units  $x_{\min}$ , and its eventual separation from the firm arrives with the rate  $\phi$  which corresponds to notice duration.

**The Firm.** Each efficiency unit of labor allows the firm to produce output using a production function  $f(k)$  which is assumed to be homogeneous of degree  $\alpha < 1$ . This implies locally decreasing returns to scale which we interpret as a limitation on the span of control. We assume that there is a perfectly competitive market for capital which is rented by the firm from households at a rental rate  $\rho$ , and that capital supply is perfectly elastic so that aggregate capital is demand-driven. The price of capital is given by  $\rho = r + \delta + \zeta$ , where  $r$  is the natural rate of discount in the economy,  $\delta$  is the depreciation rate, and  $\zeta$  is the risk premium. The firm chooses capital by equating its marginal product to its marginal cost at the efficiency unit level. Unlike [den Haan et al. \(2000\)](#), we do not explicitly model a household that saves and consumes. But, a possible way in which the

risk premium can be micro-founded in an even richer model is a stochastic shock to the households' preferences with regard to holding safe liquid assets as in the work of Fisher (2015) (i.e., a flight-to-quality shock).

The value function  $J(x, \mathbf{s})$  of the producing firm is given by

$$rJ(x, \mathbf{s}) = xp[f(k(\mathbf{s})) - \rho k(\mathbf{s})] - w(x, \mathbf{s}) + \lambda \int_{x_{\min}}^{x_{\max}} \max \{ J(y, \mathbf{s}), J^n(w(x, \mathbf{s}), \mathbf{s}) \} dG(y) - \lambda J(x, \mathbf{s}) + \tau E[\max \{ J(x, \mathbf{s}'), J^n(w(x, \mathbf{s}), \mathbf{s}') \} - J(x, \mathbf{s}) \mid \mathbf{s}], \quad (1)$$

where  $\mathbf{s}$  denotes the aggregate state of the economy,  $w(x, \mathbf{s})$  denotes the bargained wage of a worker with  $x$  efficiency units at state  $\mathbf{s}$ , and  $J^n(w(x, \mathbf{s}), \mathbf{s})$  denotes the value of being in a state of notice. The firm discounts its production profits by  $r$  and takes into account the possibility of two shocks, a match-specific idiosyncratic shock with arrival rate  $\lambda$  after which the firm will choose whether or not to stay matched with the worker or to give notice of separation, and an aggregate shock that arrives with hazard-rate  $\tau$  which embodies the same choice.<sup>4</sup> If notice was given, the wage level is fixed at  $w(x, \mathbf{s})$  and cannot be updated. Thus, the value of a firm during the state of notice is given by:

$$rJ^n(w(x, \mathbf{s}), \mathbf{s}) = -w(x, \mathbf{s}) + x_{\min} p[f(k(\mathbf{s})) - \rho k(\mathbf{s})] + \phi(V(\mathbf{s}) - J^n(w(x, \mathbf{s}), \mathbf{s}) - Fpf(k(\mathbf{s}))) + \tau E[J^n(w(x, \mathbf{s}), \mathbf{s}') - J^n(w(x, \mathbf{s}), \mathbf{s}) \mid \mathbf{s}], \quad (2)$$

where  $\phi$  is the hazard-rate associated with the arrival of a firing-permission and ending the notice period,  $V(\mathbf{s})$  is the value of a vacancy, and the firing cost is  $Fpf(k)$ . When viewed from the point of view of the individual firm, it is more convenient to think of

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<sup>4</sup>We consider a change in the aggregate state as a re-draw of certain model parameters from a discrete known state-space. To economize on notations, we do not denote the state-dependence of each parameter, thus facilitating generality and avoiding cumbersome notations such as  $\rho(\mathbf{s})$ .

$Fpf(k)$  as a tax on separation rather than an output loss cost. However, from the point of view of the aggregate firm, which will be constructed later, we will interpret  $Fpf(k)$  as an output loss cost and not as a tax. The main reason for this is that we think of  $Fpf(k)$  as a non-pecuniary adjustment cost at the aggregate level and not as a tax with re-distributive effects. The firing cost is a cost in lost output, by way of using existing and paid for labor and capital in the effort of firing a worker. This, in reality, would consist of paying a lawyer, meetings with unions, conducting a hearing before the notice is given, and so on. This makes  $F$  into the number of efficiency units of labor that must be spent in such a process and  $Fpf(k)$  into a quantity in terms of output of another job that is choosing capital optimally as in Eq. 1.

**The Worker.** Analogously, the value function for the worker  $W$  is given by

$$\begin{aligned}
rW(x, \mathbf{s}) &= w(x, \mathbf{s}) + \lambda \int_{x_{\min}}^{x_{\max}} \max \{ W(y, \mathbf{s}), W^n(w(x, \mathbf{s}), \mathbf{s}) \} dG(y) \\
&\quad - \lambda W(x, \mathbf{s}) + \tau E[\max \{ W(x, \mathbf{s}'), W^n(w(x, \mathbf{s}), \mathbf{s}') \} - W(x, \mathbf{s}) \mid \mathbf{s}], \quad (3)
\end{aligned}$$

and the value function during notice  $W^n$  is

$$\begin{aligned}
rW^n(w(x, \mathbf{s}), \mathbf{s}) &= w(x, \mathbf{s}) + \phi(U(\mathbf{s}) - W^n(w(x, \mathbf{s}), \mathbf{s})) \\
&\quad + \tau E[W^n(w(x, \mathbf{s}), \mathbf{s}') - W^n(w(x, \mathbf{s}), \mathbf{s})], \quad (4)
\end{aligned}$$

where  $U(\mathbf{s})$  is the value from being in a state of unemployment.

**Bargaining and The Separation Choice.** As is standard in the search and matching literature, the wage is given by a continuous-time Nash bargaining problem. In our model, the wage bargaining is slightly more conceptually challenging because of the presence of termination notice. The introduction of termination notice imposes that the bargained wage will be the wage during the advance notice period and in essence, makes the outside option of each side dependent upon the wage. We will show that this dependence is not problematic in our setup and that we do not need to keep track of the wage itself to obtain all of the model dynamics for job-creation and destruction. The key intuition behind this result is that as long as the bargaining problem is still a transferable utility game, any mandated transfer can be offset by the bargaining mechanism via changing the wage.

The only reason for a pair to change their working arrangement by changing the wage or separating is a re-draw of the aggregate or the idiosyncratic state. Without changing these, each existing match will keep on going forever and no separations and termination notices will occur. Thus, we present two bargaining problems. The problem of a continuing pair, which is given by:

$$w(x, \mathbf{s}) = \arg \max (W(x, \mathbf{s}) - W^n(w(x, \mathbf{s}), \mathbf{s}))^\beta (J(x, \mathbf{s}) - J^n(w(x, \mathbf{s}), \mathbf{s}))^{1-\beta}, \quad (\text{B1})$$

and that of the updating pair

$$w^*(x^*, \mathbf{s}^*) = \arg \max (W(x^*, \mathbf{s}^*) - W^n(w(x, \mathbf{s}), \mathbf{s}^*))^\beta (J(x^*, \mathbf{s}^*) - J^n(w(x, \mathbf{s}), \mathbf{s}^*))^{1-\beta}, \quad (\text{B2})$$

where the wage is updated from its previous level  $w(x, \mathbf{s})$ , which was the result of (B1) under the state  $(x, \mathbf{s})$ , given a new state  $(x^*, \mathbf{s}^*)$ . The key difference between (B1) and

(B2) is that in (B1) the wage affects the outside option, while in (B2), the outside option is fixed.

**Lemma 2.1.** *The choice of separation has the Markov property. The bargaining problems (B1) and (B2) are governed by the same surplus level  $M(x, \mathbf{s})$ , and separation depends only on the current realization of  $(x, \mathbf{s})$  and not on wage history.*

*Proof.* The two problems have the following standard first order conditions:<sup>5</sup>

$$\beta(J(x, \mathbf{s}) - J^n(w(x, \mathbf{s}), \mathbf{s})) = (1 - \beta)(W(x, \mathbf{s}) - W^n(w(x, \mathbf{s}), \mathbf{s})), \quad (\text{FOC1})$$

$$\beta(J(x^*, \mathbf{s}^*) - J^n(w(x, \mathbf{s}), \mathbf{s}^*)) = (1 - \beta)(W(x^*, \mathbf{s}^*) - W^n(w(x, \mathbf{s}), \mathbf{s}^*)). \quad (\text{FOC2})$$

As a result, one can define the match surplus levels for problems (B1) and (B2) correspondingly as:

$$M(x, \mathbf{s}) = J(x, \mathbf{s}) - J^n(w(x, \mathbf{s}), \mathbf{s}) + W(x, \mathbf{s}) - W^n(w(x, \mathbf{s}), \mathbf{s}), \quad (\text{M1})$$

$$M'(x^*, \mathbf{s}^*) = J(x^*, \mathbf{s}^*) - J^n(w(x, \mathbf{s}), \mathbf{s}^*) + W(x^*, \mathbf{s}^*) - W^n(w(x, \mathbf{s}), \mathbf{s}^*). \quad (\text{M2})$$

Let us define now the sum of the values during the notice as  $M_n(\mathbf{s}) = J^n(w(x, \mathbf{s}), \mathbf{s}) + W^n(w(x, \mathbf{s}), \mathbf{s})$ . Importantly,  $M_n(\mathbf{s})$  is only a function of the aggregate state  $\mathbf{s}$  and not of the wage level during the notice period. One can show this by summing together

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<sup>5</sup>The reason that these first order conditions maintain the standard form is that, as in the simple search and matching model, the bargaining with or without termination notice is a transferable utility game for two agents with the same planning horizon. As such, the following statements hold:  $\frac{\partial J(x, \mathbf{s})}{\partial w} = -\frac{\partial W(x, \mathbf{s})}{\partial w}$ , and  $\frac{\partial (J(x, \mathbf{s}) - J^n(w(x, \mathbf{s}), \mathbf{s}))}{\partial w} = -\frac{\partial (W(x, \mathbf{s}) - W^n(w(x, \mathbf{s}), \mathbf{s}))}{\partial w}$ . These derivatives are rather complicated and cancel out immediately, so in the interest of clarity we omit these cumbersome derivations from the text.

Equations (2) and (4) to obtain

$$r M_n(\mathbf{s}) = x_{\min} p [f(k(\mathbf{s})) - \rho k(\mathbf{s})] + \phi (U(\mathbf{s}) - M_n(\mathbf{s}) - F p f(k(\mathbf{s}))) \quad (5)$$

$$+ \tau E [M_n(\mathbf{s}') - M_n(\mathbf{s}) | \mathbf{s}].$$

Thus, given the aggregate state  $\mathbf{s}$ , the surplus level that corresponds to both problems is:

$$M(x, \mathbf{s}) = M'(x, \mathbf{s}) = J(x, \mathbf{s}) + W(x, \mathbf{s}) - M_n(\mathbf{s}) . \quad (6)$$

Eq. (6) means that regardless of the wage, or any other transfer structure that would be mandated during the notice period from one side to the other, the separation choice only depends on the current state  $(x, \mathbf{s})$  as long as the sum of the outside options remains unaffected.<sup>6</sup> Separation in the model would result when  $M(x, \mathbf{s}) < 0$ . Thus, for each pair, separation maintains a Markov property by virtue of being independent on past realizations.

Moreover, at the aggregate level separations in the model do not depend on the wage distribution but only on the distribution of  $x$  and the aggregate state. Since time is continuous in our setup, the aggregate wage distribution is composed only of the solutions to (B1), as the wage that solves (B2) would prevail for only an infinitesimal length of time before renegotiation according to (B1) would occur.  $\square$

**Match Surplus and The Reservation Level.** The bargaining problems above illustrate that the key determinant of separations in the model is the match surplus  $M(x, \mathbf{s})$ . We will show that for each aggregate state, there exists a minimal realization of  $x$  to which we call the reservation level, denoted by  $R(\mathbf{s})$  such that  $M(R(\mathbf{s}), \mathbf{s}) = 0$ . This means

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<sup>6</sup>Examples under which such is not the case, are cases where the wage affects the size of the surplus itself, e.g., in the presence of distortionary taxes.

that a re-draw will result in separation in state  $\mathbf{s}$  if and only if  $x < R(\mathbf{s})$ .

After some tedious but straightforward algebra one can derive the following expression for the surplus level:<sup>7</sup>

$$(r + \lambda + \tau)(M(x, \mathbf{s}) + M_n(\mathbf{s})) = xp(f(k(\mathbf{s})) - \rho k(\mathbf{s})) + \lambda \left[ M_n(\mathbf{s}) + \int_{x_{\min}}^{x_{\max}} \max(M(y, \mathbf{s}), 0) dG(y) \right] + \tau [E[\max\{M(x, \mathbf{s}'), 0\} + M_n(\mathbf{s}') \mid \mathbf{s}]]. \quad (7)$$

**Lemma 2.2.** *If there is a level of  $x$  in state  $\mathbf{s}$  such that,  $M(x, \mathbf{s})$  is positive, i.e., there is some production in state  $\mathbf{s}$ , then there exists a unique minimum level of  $x$  in state  $\mathbf{s}$  for which the match continues to produce together. This level is defined here as the reservation level of efficiency units of labor  $R(\mathbf{s})$ . The reservation level is the unique zero of  $M(x, \mathbf{s})$  at state  $\mathbf{s}$ . Any realization of  $x$  below the reservation will result in separation.*

For a formal proof, see Appendix A.2. The important part of the proof is that the function  $M(x, \mathbf{s})$  is monotonically increasing in  $x$  for each aggregate state. In the deterministic case,  $\tau = 0$  and the result is trivial with  $\frac{\partial M(x, \mathbf{s})}{\partial x} = p \frac{f(k) - \rho k}{r + \lambda}$ , the stochastic case is slightly more technically complex and is thus placed in the Appendix. A key picture to have in mind here is that the value function  $M(x, \mathbf{s})$  which is linear in the deterministic case, is in the stochastic case piece-wise linear for each state  $\mathbf{s}$ . Suppose for the sake of example that there are two possible states denoted by 1 and 2 and that without loss of generality,  $R(1) \geq R(2)$ . We can divide the support into three parts, as follows, the simplest is the interval  $[R(1), x_{\max}]$  in which an aggregate shock results in continuing the production of the pair. Thus, the option value encapsulates the probability of continuing production under an alternative state. On the interval  $[R(2), R(1))$ , a producing pair that is transitioning from state 2 into state 1 will separate immediately. Thus, the option value now

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<sup>7</sup>See Appendix A.1 for explicit step-by-step derivation of this equation.

takes into account the probability that an aggregate shock will result in separation, changing the slope of the value function compared with that in the previous interval. The last interval  $[x_{\min}, R(2))$  has no producing realizations in it and the value function is negative in each state. Thus, if there are  $a$  possible states, the function  $M(x, \mathbf{s})$  is piece-wise linear with  $a$  break points and  $a + 1$  intervals. In every one of these intervals the value function is linear with an exact slope described in Appendix A.2.

**Job-Creation.** Firing restrictions will affect the hiring decision by changing the expected value of the resulting match. However, by design the restrictions will only be applied to existing matches and not to newly forming ones. At the point of meeting, the employee and the employer, have as their outside options unemployment and a vacant job correspondingly, instead of the notice period that the continuing pair will face. As such, we will turn our attention now to the newly forming pair's problem or the outsiders' problem:

$$w_h(x, \mathbf{s}) = \arg \max (W(x, \mathbf{s}) - U(\mathbf{s}))^\beta (J(x, \mathbf{s}) - V(\mathbf{s}))^{1-\beta}, \quad (\text{B3})$$

where the wage of the newly hired worker will be  $w_h(x, \mathbf{s})$  with  $U(\mathbf{s})$  denoting the value from unemployed and  $V(\mathbf{s})$  is the value of a vacancy.

Unlike the bargaining problems discussed earlier, the problem (B3), is quite standard as the outside option for each side does not depend on the wage. In the literature, one sometimes encounters this type of outsider problem with a different firm value than  $J(x, \mathbf{s})$ , called for example  $J^h(x, \mathbf{s})$ . This is done because the hiring wage is  $w_h(x, \mathbf{s})$  and not  $w(x, \mathbf{s})$ . Importantly, at the moment after hiring is done, the wage will be renegotiated and the bargaining would result in the wage level  $w(x, \mathbf{s})$ . Thus,  $J(x, \mathbf{s})$  and  $J^h(x, \mathbf{s})$  are the expected values of two financial assets yielding identical expected div-



ident streams other than the divided at the first period. In a discrete-time model, this difference is important and must be taken into account. However, in a continuous-time model, the difference in wages takes place only for an infinitesimal amount of time and  $J(x, \mathbf{s})$  and  $J^h(x, \mathbf{s})$  are equal to one another as a single point-wise discontinuity does not alter the value of an integral. The same case can be made for the household.

As in the previous problems, we can understand the problem by examining the match surplus associated with (B3) which is:

$$M_h(x, \mathbf{s}) = J(x, \mathbf{s}) + W(x, \mathbf{s}) - U(\mathbf{s}) - V(\mathbf{s}). \quad (8)$$

As is standard in search and matching models, we assume free entry which means that  $V(\mathbf{s}) = 0$  for every state  $\mathbf{s}$ . Using this assumption and Eq. (6) one can arrive at the following relationship:

$$M_h(x, \mathbf{s}) = M(x, \mathbf{s}) + M_n(\mathbf{s}) - U(\mathbf{s}). \quad (9)$$

Thus, the relationship between the surplus at hiring and the surplus in continuation is linear and depends upon the values of the gained notice period and forgone unemployment  $M_n(\mathbf{s}) - U(\mathbf{s})$  which can be larger or smaller than zero given model parameters.<sup>8</sup>

Importantly, since  $M_h(x, \mathbf{s})$  depends on  $x$  only via  $M(x, \mathbf{s})$ , their derivatives with respect to  $x$  are equal. Thus, at each state  $\mathbf{s}$ , there is a unique zero for  $M_h(x, \mathbf{s})$ , but given that  $M_n(\mathbf{s}) - U(\mathbf{s})$  is not necessarily equal to zero, the solution to  $M_h(x, \mathbf{s}) = 0$  is probably different than  $R_s$ .

At this point, one's assumptions regarding information play a key role. If one assumes

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<sup>8</sup>For example, in the deterministic case  $\tau = 0$  with  $x_{\min} = 0$  we have that  $(r + \phi)M_n(\mathbf{s}) = \phi(U(\mathbf{s}) - Fpf(k))$  which makes  $M_n(\mathbf{s}) < U(\mathbf{s})$  as  $Fpf(k)$  is positive and  $\phi$  is positive. One can choose a higher value of  $x_{\min}$  that would alter this result for some or all states.

that  $x$  is known exactly at the time of the meeting, the model has two reservation levels, one above which hiring occurs and one below which termination occurs. There are three problems with this assumption: First, it is very unrealistic and unlikely that a pair will know following an instant's interaction the exact quality of their respective match as we are dealing with a single search pool; Second, a quantitative implication of having two reservation levels, which will be illustrated in detail when we discuss the calibration, is that the distribution of  $x$  among existing matches, and the wage distribution, as a result, will have a very distinct kink at the higher of the two reservation levels. This is an unrealistic feature for an earning distribution; Third, on a technical level, the model becomes more difficult to handle and it is harder to draw analytical conclusions. If one assumes no knowledge of  $x$  at the point of hiring, hiring is solely based upon expectations of  $x$  which will be identical for every pair. This assumption is very analytically appealing but it has a very problematic feature. A large amount of newly hired workers will be fired on their first day as in the absence of information, hiring will occur at  $x < R(\mathbf{s})$ . We choose a middle-way between these two that is based on the following rationale. The no-information case is unlikely at the extreme since, in reality, firms do conduct interviews and have some sort of screening mechanism in place. Workers do some sort of screening for job openings themselves be it looking for information on-line or simply asking around about their potential employer. This screening is naturally imperfect but allows passable matches to form. We assume that the bilateral screening technology reveals to both sides whether or not their matching will be passable at the current state of the world. Technically put, the information set at the time of the meeting is symmetric and binary. Both sides receive the same information from their screening technology, which is either  $x < R(\mathbf{s})$  in which case, they continue to search, or  $x \geq R(\mathbf{s})$  in which case they choose to form a match.<sup>9</sup>

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<sup>9</sup>We abstract from strategic considerations of cut-off choice which may result in some immediate sepa-

We assume the standard Cobb-Douglas matching function  $m(u, v) = \sigma u^\eta v^{1-\eta}$  with  $\theta = \frac{v}{u}$  denoting labor market tightness. Given the screening technology, a vacant job encounters a job-seeker with rate  $q(\theta) = \frac{m}{v}$ , and a job-seeker encounters a vacant job with rate  $\theta q(\theta)$ . The vacancy-filling rate is  $q(\theta(\mathbf{s}))(1 - G(R(\mathbf{s})))$ , and the job-finding rate is  $\theta(\mathbf{s}) q(\theta(\mathbf{s}))(1 - G(R(\mathbf{s})))$ . By choosing  $\theta(\mathbf{s})$  and  $R(\mathbf{s})$  the firms control aggregate job-creation and destruction at each state.

**Value Functions During Search.** Given the matching mechanism we described, we can now discuss the final two value functions, those of the searching firm and worker. We assume that the searching firm must have some amount of capital in place, thus search cost will be proportional to the cost of capital that would be used in production. As such, the value from a job vacancy is given by

$$rV(\mathbf{s}) = -pc\rho k(\mathbf{s}) + q(\theta(\mathbf{s})) \int_{R(\mathbf{s})}^{x_{\max}} J(y, \mathbf{s}) dG(y) + \tau E[V(\mathbf{s}') - V(\mathbf{s}) | \mathbf{s}],$$

where the searching firm pays the flow cost of search by renting a proportion  $cp$ , where  $p$  is the aggregate productivity parameter, of the capital rental cost that a single efficiency unit would require  $\rho k(\mathbf{s})$  which it takes as a given. The reason that we choose this cost structure is to capture the notion that a vacancy results in some amount of idle capital with opportunity cost. With rate  $q(\theta(\mathbf{s}))(1 - G(R(\mathbf{s})))$ , the vacant job is filled with a passable worker and the pair begins to produce.

The value function can be simplified by two ways. First, we have already discussed the free-entry condition which means that  $V(\mathbf{s}) = 0$  in every state. Second, the value function  $J(x, \mathbf{s})$  is related to the hiring surplus. In particular, the first order condition of

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rations in the economy as modeling formally the screening choice is beyond the scope of this paper.

(B3), together with the free-entry condition is:

$$\beta J(x, \mathbf{s}) - (1 - \beta)(W(x, \mathbf{s}) - U(\mathbf{s})) = 0. \quad (10)$$

which means that  $J(x, \mathbf{s}) = (1 - \beta) M_h(x, \mathbf{s})$ . Taken together, the job-creation condition in the model is given by:

$$c p \rho k(\mathbf{s}) = q(\theta(\mathbf{s})) (1 - \beta) \int_{R(\mathbf{s})}^{x_{\max}} M_h(y, \mathbf{s}) dG(y) . \quad (11)$$

The value from being in a state of unemployment is given by

$$rU(\mathbf{s}) = z + \theta(\mathbf{s}) q(\theta(\mathbf{s})) \int_{R(\mathbf{s})}^{x_{\max}} [W(y, \mathbf{s}) - U(\mathbf{s})] dG(y) + \tau E(U(\mathbf{s}') - U(\mathbf{s}) | \mathbf{s}),$$

where  $z$  is the flow value of being unemployed. Using Eq. (10), which implies that  $W(x, \mathbf{s}) - U(\mathbf{s}) = \beta M_h(x, \mathbf{s})$  we finally obtain:

$$\begin{aligned} rU(\mathbf{s}) &= z \\ &+ \theta(\mathbf{s}) q(\theta(\mathbf{s})) \beta \int_{R(\mathbf{s})}^{x_{\max}} M_h(y, \mathbf{s}) dG(y) + \tau E(U(\mathbf{s}') - U(\mathbf{s}) | \mathbf{s}) \end{aligned} \quad (12)$$

The model solution can be completely characterized by solving the system of equations that is composed of (5), (7), (8), (11), and (12). Numerically speaking, the integral expression will be solved using integration by parts or discretization, which will be discussed in Section 3.

## 2.2 Aggregation.

**Population Composition.** The population size is normalized to unity and it is composed of three groups: unemployed persons  $u$ , employed persons  $e$ , and those employed under termination notice  $n$ , where  $u + e + n = 1$ . Given the previously described mechanisms for hiring and terminations, aggregate termination will depend on the distribution of  $x$  across the productive realizations. This distribution, which we denote by  $H(x)$ , has the following law of motion:

$$\begin{aligned} H_{t+1}(x) e_{t+1} = & e_t (H_t(x) - H_t(R_{t+1})) + \theta_{t+1} q(\theta_{t+1}) u_t (G(x) - G(R_{t+1})) \\ & + \lambda e_t [(1 - H_t(x))(G(x) - G(R_{t+1})) - (H_t(x) - H_t(R_{t+1}))(1 - G(x))] \\ & - \lambda e_t (H_t(x) - H_t(R_{t+1})) G(R_{t+1}), \end{aligned} \quad (13)$$

where  $t$  and  $t + 1$  denote time. We shift momentarily into discrete-time notation as we find that it helps to clarify the non-linear dynamics of the model. The first term in the right-hand side relates to the immediate outflow from employment into notice that results when the reservation level increases. Note that when the reservation remains unchanged or decreases  $H_t(R_{t+1}) = 0$  since there were no producing realizations below  $R_{t+1}$  during period  $t$ . If the reservation were to increase, we would observe a positive amount of separations of mass  $e_t H_t(R_{t+1}) > 0$ . The second expression represents inflow into employment at  $x$  or below it. The last three terms represent changes in the distribution that result from idiosyncratic shocks that result in, lowering  $x$ , increasing  $x$ , or separation of active matches respectively.

Together with this law of motion, the population dynamics in the model can be characterized by the following laws of motion for the three masses:

$$u_{t+1} = u_t - \theta_{t+1} q(\theta_{t+1}) (1 - G(R_{t+1})) u_t + \phi n_t, \quad (14)$$

$$e_{t+1} = e_t + \theta_{t+1} q(\theta_{t+1}) (1 - G(R_{t+1})) u_t - [\lambda G(R_{t+1}) (1 - H_t(R_{t+1})) + H_t(R_{t+1})] e_t, \quad (15)$$

$$n_{t+1} = n_t - \phi n_t + [\lambda G(R_{t+1}) (1 - H_t(R_{t+1})) + H_t(R_{t+1})] e_t. \quad (16)$$

**Population Composition in The Deterministic Model.** We find that it is useful to observe, for the sake of the discussion that follows, the steady-state values for the masses and for the distribution  $H(x)$ . The deterministic steady-state population masses are:

$$\begin{aligned} u &= \frac{\phi \lambda G(R)}{\theta q(\theta) (1 - G(R)) (\phi + \lambda G(R)) + \phi \lambda G(R)}, \\ e &= \frac{\phi \theta q(\theta) (1 - G(R))}{\theta q(\theta) (1 - G(R)) (\phi + \lambda G(R)) + \phi \lambda G(R)}, \\ n &= \frac{\lambda G(R) \theta q(\theta) (1 - G(R))}{\theta q(\theta) (1 - G(R)) (\phi + \lambda G(R)) + \phi \lambda G(R)}. \end{aligned}$$

For the distribution, one can use Eq. (13) to obtain that

$$H(x) = \left[ \frac{u}{e\lambda} \theta q(\theta) + 1 \right] (G(x) - G(R)),$$

and by substituting in the values for the steady-state masses we obtain

$$H(x) = \frac{G(x) - G(R)}{1 - G(R)}. \quad (17)$$

Viewed from the perspective of  $R$  as the cutoff level for the screening technology, this is simply Bayes' rule applied to  $G(x)$ , i.e., the deterministic steady-state of  $H(x)$  consists

of all the possible realizations of  $G(x)$  conditional upon them being passable ones. In the stochastic case this stylized result slightly breaks down because we are dealing with long-term expectations. As a result of the fact that there are several levels of  $R$ , one for each aggregate state, leading to some discontinuities in the long-term expectations for the distribution. However, the deterministic case is very much instructive for understanding the relationship between  $R$  and  $H(x)$ . The reservation level  $R(\mathbf{s})$  is the realized lower bound of  $H(x)$  in state  $\mathbf{s}$ , and it affects the density of productive realizations at each value of  $x$ . The higher the value of  $R$  the more density is concentrated at each level of  $x$  above  $R$ .

**Aggregate Quantities.** Aggregate output  $Y$  in our model is given by:

$$Y = pf(k) \left[ e \int_R^{x_{\max}} x dH(x) + nx_{\min} \right] - Fpf(k)\phi n, \quad (18)$$

where  $k$ , as before, denotes the level of capital chosen at the efficiency unit level,  $pf(k)$  is then production per efficiency unit of labor and  $e \int_R^{x_{\max}} x dH(x) + x_{\min}n$  is the aggregate amount of such units. The last term is the output-loss cost associated with the final termination of employment relationship at the end of the notice period. The cost  $Fpf(k)$  is paid for the outflow of terminated employees which is  $\phi n$ .<sup>10</sup> Although we omit time and state notations, this expression is true globally. Importantly, the level of output depends not only upon the capital choice and the aggregate labor input used in production  $L = e + n$ , but it depends upon the current composition of  $L$ , via the sizes of  $e$  and  $n$ , the distribution  $H(x)$ , and the adjustment cost parameter  $F$ . Economically speaking, output in this economy depends upon the amount  $e$  and the quality  $H(x)$  of actively producing matches, upon the amount of matches under termination notice  $n$ , their production value

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<sup>10</sup>One can alternatively define these costs as associated with the inflow into notice, the results and interpretation that follow remain unaltered in any significant way.

and the cost of their termination.

It is more convenient to examine output using the following expression

$$Y = pf(k) \left[ e \int_R^{x_{\max}} x dH(x) + n(x_{\min} - F\phi) \right],$$

from which we can define the effective distribution of efficiency units in the economy as:

$$H_E(x) = \begin{cases} \frac{e}{n+e} H(x) & \text{if } R \leq x \leq x_{\max} \\ \frac{n}{n+e} & \text{if } x = x_{\min} - F\phi \end{cases}. \quad (19)$$

This equation means that one can interpret the composition of aggregate labor  $L$  as having an efficiency units distribution  $H_E(x)$ . This distribution has an atomistic mass that depends on the number of workers under notice out of the aggregate labor  $\frac{n}{n+e}$  at  $x = x_{\min} - F\phi$  efficiency units which is lower than  $x_{\min}$  as a result of the termination costs leading to output loss.

Along the same line, one can define the aggregate effective capital as the sum of capital over all effective producing efficiency units which is

$$K_E = k \left[ e \int_R^{x_{\max}} x dH(x) + n(x_{\min} - F\phi) \right]. \quad (20)$$

**Aggregate Production.** With the previous notations at hand, let us finally examine the aggregate production function in the economy. Let  $\bar{x}_E$  be the mean of the efficiency-unit distribution  $H_E(x)$ , from Eq. (19) we obtain

$$\bar{x}_E = \left[ \frac{e}{n+e} \int_R^{x_{\max}} x dH(x) + \frac{n}{n+e} (x_{\min} - F\phi) \right]. \quad (21)$$



We can use this to further simplify Eq. (20) as simply  $K_E = k\bar{x}_E L$ , and Eq. (18) can be reduced into  $Y = p f\left(\frac{K_E}{\bar{x}_E L}\right) L\bar{x}_E$  or by utilizing the homogeneity of degree  $\alpha$

$$Y = p\bar{x}_E^{1-\alpha} K_E^\alpha L^{1-\alpha}, \quad (22)$$

this expression omits the time and state dependence of  $\bar{x}_E$  and of the factors for brevity but holds globally.

### 2.3 Discussion of The Model and Its Implications for Aggregate Cyclical Behavior

The model is a partial equilibrium model in which firms choose optimally capital and hire labor in a heavily frictional environment. Some of these frictions are natural, and some institutional. Search frictions are a natural feature of the labor market as there is an inherent need for time and information to search for labor. High quality of institutions and infrastructure may alleviate some of the costs and challenges associated with the search for labor, but the essential need for search is natural. Institutional frictions, however, are those that arise directly from the institutional setup in place and can be altered by policy-makers.

Firing restrictions are a particular instance of these institutional frictions. Termination notice and red-tape costs associated with firing lead to reduced productivity and may interact with the business cycle in a fashion that amplifies its effects. In what follows we will illustrate these statements using our model.

**Productivity in The Model.** At first glance, Eq. (22) is the classical Cobb-Douglas production function with the productivity process given by  $p\bar{x}_E^{1-\alpha}$ . The parameter  $p$  is a match-level exogenous productivity parameter. However,  $\bar{x}_E^{1-\alpha}$  is endogenous, it arises

from the model fundamentals, and can be interpreted in several ways. One interpretation of  $\overline{x_E}^{1-\alpha}$  is that of a labor-augmenting technology process. This expression is a part of the productivity process that is linked to the labor input and arises directly from the quality of the workers that compose  $L$ , the aggregate labor in our economy. However, this is not a growth model, and if we consider the fact that this expression is time-dependent, we would see that it has a stationary behavior that is given by model parameters. Thus, this interpretation, though appealing, provides us with little insights.

What is  $\overline{x_E}$ ? The value of  $x$  at the match level is proportional to the marginal product of labor. The marginal product of one worker in the economy is  $x p f(k)$ , the match-level output. Viewed through the lens of the seminal works of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), one can interpret the dispersion of  $x$  as factor misallocation, i.e., dispersion in the marginal products of factors of production across firms and establishments. Therefore, if one takes the growth accounting interpretation of Eq. (22), one can obtain:

$$\hat{Y} = \underbrace{\hat{p}}_{\text{Technology}} + \underbrace{(1-\alpha)\hat{x}_E}_{\text{Labor misallocation}} + \underbrace{(1-\alpha)\hat{L} + \alpha\hat{K}_E}_{\text{Factor quantities}} \quad (23)$$

where  $\hat{Y}$  denote the log change from steady-state levels. This endogenous and time-varying TFP as a result of misallocation is analogous to the aggregation result in Moll (2014), but there the source of frictions is situated in the capital market and not the labor market.

**Under-Utilization vs. Misallocation.** If one takes the simplistic Solow-residual approach to measuring TFP changes, one would find that its results would differ from our TFP term  $p\overline{x_E}^{1-\alpha}$  due to the difference between aggregate capital and aggregate effective capital. This sensitivity to capital utilization is a known problem of such measurements

that more sophisticated measurement techniques are trying to account for, e.g., [Basu et al. \(2006\)](#). One could argue that there should be an adjustment in our model for underutilization of labor that comes from the termination notice mechanism. Thus, one could claim that our TFP expression is also not utilization adjusted and that the second term in Eq. (23) should be further broken down into a labor under-utilization term and a labor misallocation term. We argue that this is not the case and that the distinction arises from a fundamental conceptual difference.

Under-utilization implies choice. The firm can make more use out of its factors of production and chooses as an endogenous choice arising from internal costs or constraints not to do so. The firm could have some option value from this under-utilized capital, keeping the capacity for a future increase in utilization if the conditions merit such a change in optimal behavior. Misallocation, however, is a deviation from the first-best allocation that arises from market conditions. If a benevolent social planner could re-allocate the worker from a state of notice to a state of employment, or from a state of unemployment to a state of employment, the planner would do so to increase welfare. However, the frictions prevent the market from achieving this result on its own. This distinction is the reason we choose to interpret  $\bar{x}_E$  as a misallocation term.

Our aggregation result can prove useful for future research focused on comparing productivity and growth across countries and sectors of the economy. Eq. (23) and Eq. (21) can be combined in a decomposition exercises aimed at constructing TFP series for international comparisons of productivity which take into account differences in the institutional setup across different countries and sectors.

**Steady-State TFP.** Let us begin by examining the simplified economy with no firing restrictions at all. In that economy  $F = 0$  and  $\phi \rightarrow \infty$ , thus we have simply  $L = e$ , and  $n = 0$ . If we define the mean of  $H(x)$  as  $\bar{x} = \int_R^{x_{\max}} x dH(x)$ , we have that  $\bar{x}_E = \bar{x}$  or that

$\bar{x}_E$  is proportional to the average of the marginal products of labor. We call this limited case the flexible economy, and our general case the restricted economy, comparing the value of  $\bar{x}_E$  in the two cases yields that:

$$\bar{x}_E^{\text{restricted}} = \frac{e}{n+e} \bar{x}^{\text{restricted}} + \frac{n}{n+e} \left( x_{\min} - \frac{F\phi}{p f(k)} \right) < \bar{x}^{\text{flexible}} = \bar{x}_E^{\text{flexible}} .$$

If one supposes that  $\bar{x}$  is the same for both cases, this is a trivial result as  $\bar{x} > x_{\min}$ . The insight which strengthens this result is that  $\bar{x}^{\text{restricted}} < \bar{x}^{\text{flexible}}$ , why is that? The introducing firing restrictions to the economy does not change the production value of a job, but, it changes the outside option. When separation is costly, the outside option is worse and the match surplus increases for each level of  $x$ . Thus, firing restrictions lower the reservation level which is the lowest admissible realization of  $x$ . This result, the decrease of  $\bar{x}$  is the key result of Lagos (2006),<sup>11</sup> so we do not treat it formally in this paper as our focus is on the cyclical element of misallocation. However, this merits a short discussion of the relationship between the two models.

Our modeling approach owes much to Lagos' work, and the main result in Lagos (2006), the dependence of steady-state productivity and in the reservation level and the institutions in place is preserved in our model, there are several key differences. First, we view our model as a generalization of Lagos (2006) that allows for some additional elements, namely endogenous capital choice at the job level, aggregate stochastic shocks and, allowing for policies under which the surplus for the hiring and firing decisions is not the same.<sup>12</sup> Second, We consider slightly different policies, as Lagos does not allow for termination notice, and our interpretation for layoff costs is that of output loss and not

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<sup>11</sup>See Theorem 2 in Lagos (2006).

<sup>12</sup>This simplifying assumption in Lagos (2006) is not broadly discussed in the paper itself but is discussed at some length in its appendix. The arguments that Lagos presents for why this limitation is not a major one in the setup he considers is that he mainly thinks of layoff taxes and hiring subsidies for which this is not necessarily problematic (see footnote no. 50 in Lagos (2006) appendix). In our institutional setup, this argument no longer holds because of the existence of termination notice.

that of taxes. This choice affects the aggregation results.

In Lagos' model, the only way by which policies affect productivity is by changing the reservation level. In our model this channel is present, but it is not the only one, and later we will also show that this is not the quantitatively dominant one for cyclical implications. Our firing restrictions lead to an output loss even given the same reservation level and the notice mechanism causes a further decline in aggregate productivity. What about the cyclical implications?

**Implications For Cyclical Behavior.** If we consider a recession as a decline in the aggregate productivity parameter  $p$ , or as an increase in the price of capital  $\rho$  via the risk premium  $\zeta$ , the result is that a recession is a period during which the marginal product of labor  $xpf(k)$  is reduced. As such, the match surplus for a given level of  $x$  decreases and the reservation level would increase as a result. As this is the lowest level possible for a producing realization, the economy with no firing restrictions would see an increase in  $\bar{x}$  as a result. Thus generating, a 'cleansing effect' of the business cycle through an increase in the quality of labor, that would manifest empirically as rising TFP. However, an increase in the reservation leads to more separations, thus decreasing the proportion of the actively producing pairs  $\frac{e}{n+e}$  and later inducing output loss at the time of final separation. In the presence of firing restrictions, these can counteract the 'cleansing effect' and even potentially induce a 'sullyng effect', the extent of which will be explored in the quantitative and the empirical parts of the paper. In Section 3, we will show that the 'sullyng effect' arises and is the dominant effect when firing restrictions are present.<sup>13</sup>

Several key insights from the model to bear in mind for the remainder of the analysis are as follows. First, policies affect aggregate productivity in the model via three channels, (i) by affecting the population composition, (ii) by affecting the reservation level, and (iii)

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<sup>13</sup>See Barlevy (2002) for a review of these effects in the literature.

by affecting the scope of output loss caused by separations. Second, as a result of these, any cyclical adjustment in the model induces an effect on the cross-sectional distribution of  $x$  and thus influences aggregate productivity.

### 3 The Effects of Firing Restrictions on Business Cycle Dynamics - A Quantitative Assessment.

In this section, we perform a quantitative exercise aimed at illustrating the potential effects of firing restrictions on business cycle dynamics. We calibrate the deterministic steady state of the model from Section 2 to match France's job flows and institutional parameters. Our reason for choosing France is mainly one of data availability. Notably, this calibration and simulation exercise should not be viewed as an attempt to capture the complexity of France's labor market and its institutions but rather to outline the cyclical implications of the mechanisms that we have discussed in the previous section. Using the calibrated model and counter-factual institutional structures we demonstrate the propagation of an exogenous shock in the simulated economy and explore the relative importance of the different channels.

#### 3.1 Calibration

**Calibration Targets.** The model is calibrated to match France's quarterly job-finding rate of 20% and separation hazard of 3.4% based on [Hobijn and Sahin \(2009\)](#) transformed into quarterly frequency. As in [Shimer \(2005\)](#), the steady-state value of  $\theta$  is normalized to unity and  $\sigma$ , the matching efficiency parameter is calibrated to match the finding-rate. Institutional calibration is based on [Bentolila et al. \(2012\)](#), that place the replacement rate at 55%. Since our model features wage heterogeneity,  $z$  is calibrated to be 55% from the

average wage in the model economy under a deterministic steady state. Similarly, also following [Bentolila et al. \(2012\)](#),  $F$ , the firing costs are calibrated to 33% of the average quarterly output of a job.<sup>14</sup> These two hazards, two ratios and one normalization are our calibration targets. We match them exactly by choosing the values of  $\lambda, \sigma, F, z$  and  $c$ .

**Directly Calibrated Parameters.** To complete the institutional setup we build on [Bentolila et al. \(2012\)](#) and calibrate  $\phi = 0.75$  which corresponds to four months. As in [Bentolila et al. \(2012\)](#), the discount factor is set to  $r = 0.01$ , the bargaining power is  $\beta = 0.5$ , and  $\eta = 0.5$ . We normalize the common productivity factor  $p$  to yield that  $p(f(k) - \rho k) = 1$ , and set the capital share at  $\alpha = 0.33$  so that  $f(k) = k^{0.33}$ . We calibrate the depreciation rate of capital to  $\delta = 0.02$ , and the steady-state risk premium to  $\zeta = 0$ . During the simulation we will choose a mean zero process for  $\zeta$ .

**Calibration Strategy for  $G(x)$ .** The calibration of  $G(x)$  is of great importance for the cyclical behavior of the model as it dictates the nature of the option value at each state for every job. However, this calibration also presents us with a conceptual challenge. How does one observe  $G(x)$ ? The model structure imposes that only sufficiently high realizations are present in the data, i.e., we only can observe the realized distribution  $H(x)$  and not the model primitive  $G(x)$ . To add to this challenge, the only manifestation of  $H(x)$  that can be empirically observed in the data is the earning distribution. In [Appendix A.3](#), we show that the wage is linearly dependent in  $x$ . Therefore one can relate the earning distribution  $D(w)$ , to  $H(x)$  by applying a simple linear transformation whose values relate to the model parameters. This approach is very much data-intensive as one needs to have the entire earning distribution and it does not solve the truncation problem. By

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<sup>14</sup>The comparison between the setup in [Bentolila et al. \(2012\)](#) is somewhat challenging because they normalize the maximum production value to unity and assume that a new job produces the maximum amount possible. Thus, choosing the same ratio of 33% is somewhat conservative, as the average job in [Bentolila et al. \(2012\)](#) produces less than unity and the ratio is somewhat higher.

looking at the deterministic steady-state value of  $H(x)$  we can see that there is a range of realizations and their densities both of which we would never be able to observe. Specifically, how can one assume the structure of realizations that are possible in principle but would never manifest in reality? The lowest possible realization of  $G(x)$  is unknown, along with all the values and densities of  $x$  on the interval  $[x_{\min}, R)$ .

To overcome these conceptual challenges we take the following approach to calibrating  $G(x)$ . First, in the absence of a better prior, we set  $x_{\min} = 0$  which seems natural and is equivalent to the assumption that the worst worker possible is one which produces nothing. Second, we follow [Lagos \(2006\)](#) by assuming that  $G(x)$  takes a type I Pareto form with CDF:<sup>15</sup>

$$G(x) = 1 - \left(\frac{\zeta}{x}\right)^\gamma. \quad (24)$$

Why do we assume a Pareto distribution? First, and most importantly, this effectively solves the truncation problem. If we truncate  $G(x)$  at  $R$ , by using the closed-form solution for  $H(x)$  we showed in Eq. (17) we would obtain that:

$$H(x) = \frac{G(x) - G(R)}{1 - G(R)} = \frac{\left(\frac{\zeta}{R}\right)^\gamma - \left(\frac{\zeta}{x}\right)^\gamma}{\left(\frac{\zeta}{R}\right)^\gamma} = 1 - \left(\frac{R}{x}\right)^\gamma, \quad (25)$$

which makes  $H(x)$  also into a type I Pareto, with  $R$  as a scale parameter and with the same tail index  $\gamma$ . Second, and as a direct result of this, the wage distribution in the model will be, as discussed before, a linear mapping of  $H(x)$ , thus making the earnings distribution implied by the model into a skewed distribution with a power-law in the right tail and a tail index of  $\gamma$ . Not only is a skewed earnings distribution a realistic thing to consider but

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<sup>15</sup>On a purely technical level, to reconcile the above two statements, namely  $x_{\min} = 0$  and a distribution that has a minimal realization of  $\zeta$ , we substitute  $x$  in all the equations in the computations to be  $x - \zeta$  as to yield a minimum level of  $x_{\min} = 0$  and a distribution that follows a type I Pareto.



the entire distribution can be inferred from  $\gamma$ , the previously mentioned parameters and the steady-state value of  $R$ .

The last piece of the puzzle is putting a number on  $\gamma$ . Since  $\gamma$  is the tail index of a type I Pareto distribution, it bears a direct connection to the Gini coefficient of the earnings distribution. A type I Pareto distribution has a Gini coefficient of  $Gini(H) = (2\gamma - 1)^{-1}$ , where  $\gamma$  is the tail index.<sup>16</sup> Therefore, we target the Gini coefficient for the earnings distribution before transfers in France. The average Gini coefficient for the income distribution of the working-age population in France before taxes and transfers for 2012 - 2017 is 0.452.<sup>17</sup> As a result of this, we have a tail index of  $\gamma = 1.61$ .

One problem that arises in this context is comparing the empirical distribution that gave rise to the observed Gini coefficient, and the model distribution. The wage distribution in Appendix A.3 relates to the wage distribution of the actively producing realizations, while the Gini coefficient captures both producing realizations and workers under termination notice. How do we reconcile the two? It turns out that this problem is a fairly simple one and this is because of the Markov property for separations given in Lemma 2.1. Separations are independent of current realization of the wage so the wage distribution under notice  $D^n(w)$  is identical to the distribution among productive pairs  $D^e(w)$ . The distribution of wages for workers under notice evolves according to

$$n_{t+1} D_{t+1}^n(w) = D_t^n(w) n_t - \phi n_t D_t^n(w) + \lambda G(R_t) [D_t^e(w) - D_t^e(w(R_t))] e_t + D_t^e(w(R_t)) e_t,$$

where this is the exact law of motion for the mass  $n$  described in the previous section

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<sup>16</sup>For a comprehensive treatment of inequality measures and generalized Pareto distributions see Arnold (2008).

<sup>17</sup>The Gini data is taken from the OECD database at <https://stats.oecd.org/>. We choose these years as to not mix income definitions withing the OECD's database which changes the income definition after 2011. The value exhibits only small changes from year to year and the choice of time-frame will not change the results in any meaningful way.

but this time keeping track of the wage level from which entry and exit occur. In a deterministic steady state this equation reduces into  $D^n(w) = \frac{\lambda G(R)}{\phi} \frac{e}{n} D^e(w)$ , which after substituting in the steady-state masses yields  $D^e(w) = D^n(w) = D(w)$ .

The last two elements of  $G(x)$  that we have to consider are the values for  $\zeta$ , and the value for  $x_{\max}$ . The parameter  $\zeta$  is a scale parameter for  $x$  since all other calibrated targets will be either flows and hazards or ratios of the average wage or production value, we have some latitude with this parameter and so we normalize it to  $\zeta = 1$ , all other parameters will be calibrated to match that scale of the distribution. Note that we could achieve all the calibration targets listed in the next paragraph with an altered scale. Last is the value for  $x_{\max}$ . The reason we need an upper support is for the simulation exercise to apply integration by parts, so given the Pareto structure of Eq. (24) we simply choose  $x_{\max}$  such that  $G(x_{\max}) = 0.999$ .

**The Calibration Procedure.** As described above, we directly calibrate  $\phi, \eta, \beta, p, \delta, r, \alpha, \gamma, \zeta, x_{\min}$ , and  $x_{\max}$ . We use the aforementioned parameter values along with Eq. (5), (7), (11), and (12) while substituting in the definition of  $M_h(x)$  from Eq. (8). Since we are calibrating the deterministic steady state,  $\tau = 0$ , and the match surplus is a linear function of  $x$ . Thus, we use integration by parts and instead of Eq. (7), we use the following two equations:

$$(r + \lambda)(M(x_{\max}) + M_n) = x_{\max} p(f(k) - \rho k) + \lambda \left[ M_n + M(x_{\max}) - \frac{\partial M(x)}{\partial x} \int_R^{x_{\max}} G(y) dy \right], \quad (26)$$

$$M(x_{\max}) + (R - x_{\max}) \frac{\partial M(x)}{\partial x} = 0, \quad (27)$$

where the last equation follows from the definition of the reservation level as the zero of the match surplus and from the linearity of the deterministic case. The same form of

integration by parts is used for the integral expressions in Eq. (11), and (12) as

$$\int_R^{x_{\max}} M_h(y) dG(y) = M(x_{\max}) + (M_h - U)(1 - G(R)) - \frac{\partial M(x)}{\partial x} \int_R^{x_{\max}} G(y) dy .$$

Thus, the model solution is given by a system of five equations in the five unknowns  $R, \theta, M, M_h$  and  $U$ .

This solution gives us values for the rest of the parameter values. The calibration is given in full in column 1 of Table 1. A surprisingly good result of the model is wage dispersion which is a non-targeted moment. To target wage dispersion, our calibration of  $G(x)$  is insufficient as the other parameters, namely  $\beta, p, \alpha, \delta, \lambda, F$ , and  $r$  will affect directly the slope and the intercept of the linear mapping from  $G(x)$  into the wage distribution. If the intercept will be too large relative to the slope, variation in  $x$  one would obtain too little wage dispersion, too small and one would obtain too much. Moreover, indirectly, all other parameters will influence this as the intercept of this map is a function of the steady-state value of  $R$ .

According to Eurostat data on earnings for full-time workers in France for 2014, the earnings between the 90th and 10th percentiles 2.85.<sup>18</sup> Our calibrated model delivers a corresponding ratio of 2.70. This number is exceptionally good for this type of model, especially given the critique of wage dispersion in this class of models by [Hornstein et al. \(2011\)](#). We attribute the high wage dispersion in the model to the relatively rich modeling of labor market rigidities. In so doing, our calibration strategy delivers a technical improvement to the quantitative characteristics of this class of search models which we view as a technical contribution to this literature. However, as we will demonstrate later, this result is sensitive to the choice of the other parameters.

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<sup>18</sup>This number is based on data retrieved from Eurostat at <https://ec.europa.eu/eurostat/web/labour-market/earnings/database>, for France regarding the 10th, and 90th percentiles of the monthly earnings distribution for full-time workers.

## 3.2 Simulation

In this section we will introduce aggregate uncertainty to the model via an increase in the risk premium  $\zeta$  and explore quantitatively the effects of firing restrictions on the dynamics of the business cycle. We introduce a simple stochastic process with two states as follows:

$$\zeta = \begin{bmatrix} -0.1r \\ +0.1r \end{bmatrix}, \Pi = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}, \tau = 1. \quad (28)$$

This process can be described as the economy having two possible aggregate states. A good state in which households are willing to rent capital more cheaply at a rate of  $0.9r + \delta$ , and a bad state in which households require a larger premium on forgoing current consumption and rent capital at a rate of  $1.1r + \delta$ . The reason we use a persistent shock is that the model, like other models of this kind, lacks internal propagation as the controls  $R$  and  $\theta$  will respond immediately to a change in the aggregate state.

To isolate the role of each component of our policy set, we simulate four sets of impulse responses. All of these illustrate the convergence of the model economy from being in a bad state, state 2, to the economy's long-term expectations. The first set of impulse responses is for the baseline calibration we described above and is given in column 1 of Table 1. The last three are counter-factual calibrations having only firing costs, only termination notice or no firing restrictions at all. To eliminate the firing costs we set  $F = 0$ , and to eliminate the notice we calibrate  $\phi$  to correspond to an average duration,  $\frac{1}{\phi}$  of one working day per quarter which for France is  $\phi = 251/4$ . These counter-factual calibrations are given in columns 2, 3, and 4 of Table 1.<sup>19</sup>

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<sup>19</sup>Working with these high hazard rates necessitates simulating using very short periods. Thus, we resort to using as a unit of time in these simulations periods that correspond to 0.01 of a working quarter, i.e., for 62.75 working days a quarter and 8 working hours per working day, a period of about 5 hours.

Numerically speaking, given the significant non-linearity of the model, we exploit the property of piece-wise linearity of the value function  $M(x, \mathbf{s})$  for each state  $\mathbf{s}$  and use integration by parts at each linear segment of the function. Thus, as the calibration was done using a system of five equations, the stochastic case consists of a system of five equations per aggregate state.

**The Significance of Firing Restrictions for Business Cycle Dynamics.** The impulse responses to a temporary increase in  $\zeta$  for our baseline calibration and the three counterfactual ones described in Table 1 are presented in Figure 1. Our baseline calibration is presented in black, the counter-factual case with only termination notice in blue, the one with only firing costs in red, and the one without firing restrictions at all in green. An increase in the risk premium generates a business cycle in this economy as it generates an increase in capital cost and reduces firm profitability. The impulse responses suggest that, regardless of the calibration used, the shock results in a drop in output and employment and a rise in unemployment as we would expect in a business cycle. This cycle is characterized by firms becoming increasingly selective in their hiring practices, i.e.,  $R$  increases, and hiring intensity declines as the market becomes less tight, i.e.,  $\theta$  declines.

As discussed in the Section 2, this business cycle triggers the three channels of influence for the amount of misallocation that determines TFP in the economy. As firms become more selective, reservation levels increase which should increase  $\bar{x}$  and lead to a TFP increase. Observe the green line to see only this channel at play. Since  $H$  is a skewed distribution, its mean value is relatively unaffected by the small change at the left side of the support, and the increase in TFP is barely noticeable. If one were to add firing costs to the mix, the red line, output declines more over the cycle but productivity is nearly unaffected. The big effect takes place once one introduces termination notice. Termination notice causes TFP to decline visibly over the cycle, as is seen by the drop in TFP for

the blue line. Once the two channels are working in tandem, and termination notice is combined with firing costs, one sees an amplification of this effect.

**Shimer's Puzzle in Our Model.** There are two immediate criticisms of this simulation. First, is that the shock we introduced is a rather strong one, with an output decline of over 1.5% but the labor market effects are rather small other than for our baseline. The rise in unemployment is barely perceptible other than for our baseline and even there it is not a very strong one, and the same goes for the employment drop. Second, is that our main channel of interest, the misallocation-related TFP decline is of modest magnitude. A TFP decline of 0.139% is significant but it is not overwhelming.

The answer to these two critiques is that the one creates the other. It is a known theoretical phenomenon that the simple search and matching models do not generate sufficient amplification from shocks affecting real activity, such as a simple TFP shock, to the labor market. This phenomenon is known as Shimer's puzzle (Shimer, 2005). The TFP shock that Shimer considers, a decline in  $p$ , in a search and matching model without capital where the production value of a job is  $p$ , is isomorphic to our shock that reduces  $k$  where the production value of a job is  $pxf(k)$ . Thus, our model suffers also from insufficient amplification for the calibration shown here. In what follows we will demonstrate that it is precisely this lack of amplification that reduces the size of the TFP decline. And that given that one can generate sufficient amplification from the model this channel grows in its relative importance.

The literature on Shimer's puzzle is vast and suggests several ways of coping with such a limitation. One approach is including wage rigidities, e.g., the search and matching framework developed in Hall and Milgrom (2008) and employed in Hall (2017). However, modeling endogenous separation choice and labor market rigidities in the Hall and Milgrom (2008) framework is beyond the scope of this paper. Another possible solution

lays in the calibration, as pointed out in [Hagedorn and Manovskii \(2008\)](#), if one calibrates the bargaining power of the employee  $\beta$  to a lower value and increases  $z$ , then one would obtain stronger amplification. Under the standard Shimer-style calibration  $z$  is the replacement rate, while in an HM style calibration  $z$  is considered as the entire value of not working, which includes home production, unemployment insurance, and leisure. The argument in [Hagedorn and Manovskii \(2008\)](#) is that in a richer model, the worker should be nearly indifferent between employment and unemployment given that employment is a choice. The reason that such a calibration manages to create more amplification is that [Hagedorn and Manovskii \(2008\)](#) target the elasticity of wage with respect to productivity. This calibration, as pointed out by [Ljungqvist and Sargent \(2017\)](#), lowers the fundamental surplus in this economy, i.e., the resources available for vacancy creation by the market, which facilitates stronger amplification. Our baseline calibration does better than an economy without firing restrictions in terms of amplification as these lower the fundamental surplus in the economy. To see that this is so, mark the difference in labor market response between the black line and the green line in [Figure 1](#).

Since the [Hagedorn and Manovskii \(2008\)](#), henceforth HM, calibration strategy relies on wage elasticity, and our model features wage heterogeneity, we do not implement fully the HM calibration strategy. To the best of our knowledge, there had not been a modification of [Hagedorn and Manovskii \(2008\)](#) to endogenous separation search and matching models and developing one is beyond the scope of this paper. Instead of following the HM calibration strategy to the letter, we use their value for  $\beta = 0.052$ , the matching function  $q(\theta) = \frac{1}{(1+\theta^{\eta_{HM}})^{\frac{1}{\eta_{HM}}}}$ , and an average replacement rate of 95.5%, that corresponds to the value of  $z$  chosen in [Hagedorn and Manovskii \(2008\)](#).<sup>20</sup> Using these, we re-calibrate

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<sup>20</sup>In the [Hagedorn and Manovskii \(2008\)](#) paper the replacement rate is slightly higher since the production value  $p = 1$  and  $z = 0.955$ . To make job creation profitable the wage  $w$ , must be such that  $w < p$  so the replacement rate would be slightly higher. For the deterministic case in [Hagedorn and Manovskii \(2008\)](#), one can compute the wage as  $w = 0.9765$ , which gives a 97.8% replacement rate. This difference is not sizable.

the model using the same targets as before but with the new replacement rate and conduct the simulation exercise again along with the same counter-factual firing restrictions as before. We deviate from the HM calibration strategy by normalizing  $\theta$  to unity in the steady state. This calibration is summarized in column 1 of Table 2 and the simulation results from this exercise are presented in Figure 2.

The HM-style calibration delivers qualitatively the same results. Namely, in response to the shock output declines, unemployment rises, and employment falls. Although the output decline is not much stronger, the case with France's level of firing restriction exhibits this time a maximal decline in TFP of about 0.208%, which is about 50% stronger than the corresponding number from Figure 1. However, even larger values are quite plausible. Given that during the financial crisis, unemployment rose by significantly more than the 6% implied by the simulation. In France, unemployment rose from 7.16% in the first quarter of 2008 to 8.6% in the first quarter of 2009, which is a 20% increase. The numbers are not exquisitely large since for other countries for the same period the corresponding figures are 65% for the United States, 37% the United Kingdom, and 80% in Spain. This means that, in reality, there is considerably more labor re-allocation taking place in recessions than our simulation generates. However, our model can be very useful in understanding how such cyclical phenomena translate into a labor-misallocation-induced TFP decline.

Relating to the previous discussion of wage dispersion, this calibration, although containing the same Pareto form and same tail index  $\gamma$  is very poor in terms of wage dispersion. The ratio between the 90th and 10th percentiles is just about 1.1 which is not a very realistic fit. This is mainly because the slope of the linear mapping from  $x$  onto the wage is linearly dependent of  $\beta$  which is nearly ten times smaller in the HM-style calibration relative to our baseline.



**From an Aggregate Shock to Labor Misallocation.** What drives the cyclical decline in TFP in the model, and what determines its size? Recall that TFP in the model is given by

$$TFP = \underbrace{p}_{\text{Technology}} \underbrace{\left[ \frac{e}{n+e} \bar{x} + \frac{n}{n+e} (x_{\min} - F\phi) \right]}_{\text{labor misallocation}}^{1-\alpha}. \quad (29)$$

This expression holds at every point in time with  $\bar{x}, n, e$  changing over time. Denote the labor misallocation term by  $T$ , and for the sake of analytical convenience substitute  $F = l\bar{x}$ .<sup>21</sup> For the sake of notation clarity, time dependent values such as  $\bar{x}(t)$  denote values outside of steady state and objects without time dependence, such as  $\bar{x}$ , denote steady state values. If we log-linearize  $T$  around its steady-state

$$T = \left[ \left( \frac{e}{n+e} + \frac{n}{n+e} (x_{\min} - l\phi) \right) \bar{x} \right]^{1-\alpha},$$

we obtain that

$$T \hat{T}(t) = (1-\alpha) \frac{ne}{(n+e)} \frac{1 - (x_{\min} - l\phi)}{e + n(x_{\min} - l\phi)} (\hat{e}(t) - \hat{n}(t)) + (1-\alpha) \hat{x}(t), \quad (30)$$

where  $\hat{T}$  denotes percent deviation in  $T$  with respect to its steady-state value. To bring this equation to a more intuitive level, one can recall the laws of motion for the masses, and note that in steady-state  $n = \frac{\lambda G(R)}{\phi} e = \frac{\tau_r}{\phi} e$ , where  $\tau_r$  is the termination rate. This is not to be confused with the separation rate. The termination rate is the rate at which workers transition from being in the state of active employment into termination notice, while the separation rate is the rate of at which workers transition from being employed observably (regardless of notice status) into unemployment or  $\phi \frac{n}{n+e}$ . This notation transforms the

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<sup>21</sup>This is an innocuous transformation, the reason behind it is that the firing costs are calibrated to be a ratio of the average production value of a job that is  $p\bar{x}f(k)$ . Thus, one can substitute the firing costs of  $Fp\bar{x}f(k)$  with  $l\bar{x}f(k)$ .

above equation into

$$\hat{T}(t) = \psi(\hat{e}(t) - \hat{n}(t)) + (1 - \alpha)\hat{x}(t), \quad (31)$$

with  $\psi = (1 - \alpha) \frac{\phi\tau_r}{\tau_r + \phi} \left[ \frac{(1 - x_{\min} + l\phi)}{\phi + \tau_r(x_{\min} - l\phi)} \right]$ .

Economically speaking, the labor-misallocation-induced TFP decline we are interested in is linearly proportional to the decline in employment and the increase in termination notices. Under our baseline calibration, the value of  $\psi$ , the parameter that dictates the change in the misallocation term of TFP, is about  $\psi = 0.0383$  for both calibrations. However, since the steady-state mass of workers under notice ( $n$ ) is small, a fractional change in its value is a large change in  $\hat{n}(t)$ , and this will be the dominant effect. To illustrate the potential strength of this effect, suppose that the termination rate goes up by 10%. This may be a very small change in employment since we are considering a change in the flow out of active employment which may be quite slow, but it would translate to a 0.383% productivity decline, which is a sizable figure.

In Figure 3, we present an illustrative sensitivity analysis of  $\psi$ . Each panel in Figure 3 presents the values of  $\psi$  given our baseline calibration while changing only two parameter values. Hazard rates are again in quarterly frequencies. Although we do not explore each possible combination of parameter values, the main take away is that the following aspects can amplify the TFP decline in our economy: An increase in the labor share  $1 - \alpha$ ; an increase in the frequency of labor turnover  $\tau_r$ , i.e., a decrease in average job duration; an increase in the length of the notice period  $\frac{1}{\phi}$ ; and an increase in the cost of separation  $l$ .

To conclude, this quantitative exercise illustrates the potential of firing restrictions to act as an amplifier of macroeconomic shocks. The presence of strict firing restrictions in the baseline calibration leads to more misallocation of labor in response to a shock. As a result, TFP declines more severely and persistently due to increasing amounts of labor misallocation over the cycle. Our simulations suggest that the strength of the effect on

TFP depends upon the magnitude of the employment decline and the increase in the number of workers under notice which, being a percentage change of a relatively small number, can be quite large. With these implications in mind, we now turn our attention to the empirical analysis of EPL. Our empirical analysis will examine the effect of EPL, which will examine the transmission of global credit supply shocks into real activity and the labor market.

## 4 Data

### 4.1 Measurement of Firing Restrictions

EPL is measured as a ‘hierarchy of hierarchies’, meaning it is the aggregate of several scales which rank the strictness of legislation (e.g., from 0 to 6 as in the OECD’s indices), where these scales are aggregated according to predetermined weights.<sup>22</sup>

EPL is a broad institution and the OECD’s database of EPL includes several indices measuring it which differ in terms of their coverage and of their implications:<sup>23</sup> regular employment protection, protection from collective dismissals, and protection of temporary workers. These indices, though grouped under the label of employment protection, relate to different segments of the labor-force, i.e., regular workers and temporary workers and apply to different circumstances. Thus, these legislative measures affect the dynamics of labor turn-over through different channels.

The first noteworthy difference is that between the protection of regular and temporary workers. Protection of regular employees includes the definition of wrongful termination, the procedure of terminating an individual employee, severance pay and notice

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<sup>22</sup>A critique of this measurement method and its limitations can be found in [Myant and Brandhuber \(2016\)](#).

<sup>23</sup>A more comprehensive discussion of EPL measurements, coverage, and definitions can be found in [Boeri and van Ours \(2013\)](#).

due, and the legal recourse available to a wrongfully terminated worker. However, protection for temporary workers includes just cause of hiring under a temporary contract, the verities of jobs a person thus contracted may perform, when can temporary work agencies be used, and the number of successive temporary contracts per worker. In other words, the protection of regular workers is a firing restriction, while the protection of temporary ones is a hiring restriction. Although the two are quite different, limitation on each form of employment makes the other form relatively more attractive for employers, so the interaction between the two forms of protection had received substantial focus in the literature and policy debate in recent years.<sup>24</sup>

The second is protection from collective or individual dismissals. The protection of regular employees relates to the case of individual termination. In the case of collective termination of workers due to rescaling, reorganization, or other changes in the firm-level other protection measures govern such a procedure.<sup>25</sup>

Since our main interest in this paper is the effects of firing restrictions, we use the index 'Strictness of employment protection - individual dismissals (regular contracts)' (EPR V1) as our measure of firing restrictions.<sup>26,27</sup> For brevity's, for the remainder of this paper, the term EPL refers only to this form of protection. When we refer to other employment protection policies those policies are explicitly named.

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<sup>24</sup>See Boeri and Garibaldi (2007), Cahuc et al. (2016) for a thorough discussion on the substitutability of the two employment forms, and Bentolila et al. (2012) for a case study of these two protection forms in Spain and France and an overview of the policy debate on the subject. For works dealing with the cyclical effects of protection of temporary measures see, among others, Bentolila and Saint-Paul (1992), Nunziata and Staffolani (2007), and Cahuc et al. (2016)).

<sup>25</sup>Examples of such legislative measures can be found in *EUR-Lex Directive 98/59*. (2016).

<sup>26</sup>See Table 3 for the break-down of the index to its components and the data that composes each.

<sup>27</sup>Data for this EPL index is also available annually for 1960-2004 in a database created by Nickell (2006). However, the index displayed there for the years 1960 to 1985 is a backward extension of the OECD's index created by assuming that its rate of change over time is the same as the change in another index which uses data taken from Blanchard and Wolfers (2000) and from Lazear (1990). From 1985 onward the index provided by Nickell (2006) is the same as the OECD's index. Since the OECD's index is available for twenty-eight consecutive years for most of our sample, we chose, for the sake of consistency, to rely on the OECD's index instead of utilizing a mixed measurement methodology.

Although our focus is on the protection of regular employees from individual dismissals, other forms of employment protection and other labor market institutions are present alongside our main institution of interest. Taking this into consideration, we use data on other forms of employment protection and labor market institutions in our robustness analysis in Online Appendix B to assure that our results are not confounded by other institutional factors.

EPL indices are composed of several scores which are ordered variables. The final index can take non-integer values, as can the individual components, but that does not change the fact that the components themselves are a ranking system of ordinal variables. This point stresses the importance of using an identification strategy that allows for variation in an ordered variable and not in a continuous one. We choose to use dummy variables to identify policy regimes rather than take the index's levels. This order-preserving identification approach avoids manipulations to the ranking scale that can result from using continuous interactions with the index. Specifically, one could conceive of an order-preserving non-linear transformation of the EPL components which would reflect the same order of ranking but would change the results of a continuous-interaction-based regression analysis. Nevertheless, the conventional treatment of the EPL index has largely been as if it were a continuous variable. Noteworthy examples of this can be found in [Blanchard and Wolfers \(2000\)](#), [Messina and Vallanti \(2007\)](#), [Nunziata \(2003\)](#), and [Duval and Vogel \(2008\)](#). The only methodological exceptions to this, to the best of our knowledge, are studies which consider only the cardinal elements of EPL such as months of notice and months of payment offered as severance pay and ignore the regulatory environment as in [Lazear \(1990\)](#), or studies that focus on correlations and utilize Spearman correlation coefficient as in [Gnocchi et al. \(2015\)](#).

## 4.2 Outcome Measures

In order to examine the implications of EPL for macroeconomic resilience, we have created a panel containing the following variables:<sup>28</sup> Key labor market variables: unemployment, employment to population ratio, and labor force participation rates; National accounts data (all in real terms): output,<sup>29</sup> consumption, investment, government expenditure, imports, and exports; TFP; capacity utilization; total hours worked; hours worked per worker; our shock variable, EBP, which will be discussed shortly; and our state variable, the EPL index. We use data from 21 OECD economies for the period between 1985 to 2013.<sup>30</sup> Our choice of sample, both along the country dimension and the time dimension, arises from the availability of the EPL index.<sup>31</sup>

Our dependent variables are taken from the OECD's database.<sup>32</sup> All dependent variables are taken as log cumulative changes on the LHS of the regressions and as log-first-differences when controlled for in lags on the RHS of the regressions. We use the dependent variables in log cumulative changes to properly compare movements in a variable between different countries with different steady-state levels.

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<sup>28</sup>For further details and information on the data used in this paper see Online Appendix C.

<sup>29</sup>We use output and not output per-capita for two reasons: First, to be consistent with the other national accounts data that are available only without such normalizations; and second, due to data availability, for output, we have 2,068 quarterly observations while for output per-capita only 1,774 such observations are available for the same countries and time-frame. In Online Appendix B we show that our results are robust to using this choice of measure.

<sup>30</sup>We use monthly data for unemployment and quarterly data for the rest of our variables of interest; all data are seasonally adjusted except EPL which is available only in annual frequency and assumed identical within each year.

<sup>31</sup>In the UK the OECD's EPL index is available for 2014 and therefore we use data from this year as well for the UK.

<sup>32</sup>All OECD data were retrieved from <http://stats.oecd.org/>; for exact details see Online Appendix C.

### 4.3 Shock Variable

As the shock variable in the analysis that follows we will use the Excess Bond Premium (EBP) measure from [Gilchrist and Zakrajšek \(2012\)](#), who use micro-level data to construct a credit spread index which they decomposed into a component that captures firm-specific information on expected defaults and a residual component that they termed as the excess bond premium. To the best of our knowledge, there is no financial shock variable which was calculated specifically for every one of the economies we use in our analysis. That said, the increasingly global nature of the world economy means that EBP can be interpreted as a global shock variable whose effects on the economies in our sample can potentially vary as a function of the EPL regime in place.

## 5 Methodology

We follow the class of specifications that use the local projection method from [Jorda \(2005\)](#) to estimate impulse response functions and adapt it to a state-dependent setting as the one employed in [Auerbach and Gorodnichenko \(2012\)](#), [Owyang et al. \(2013\)](#), [Ramey and Zubairy \(2017\)](#), and [Tenreyro and Thwaites \(2016\)](#). The major advantage of this identification method is that it allows for state-dependent non-linear effects in a straightforward manner while involving estimation by simple regression techniques. Moreover, it is more robust to misspecification than a non-linear VAR. Additionally, it can be used to analyze data of differing measurement frequencies as one is not required to estimate the system in a joint fashion.

**Definition of EPL States.** In defining the state of EPL we wish to group observations in a way that allows for sufficient differentiation to be made between the groups and in a manner that can describe broadly the policy regime in place; too many groups will limit

sample sizes severely, while too few will not enable differentiation. To allow for sufficient differentiation, we use the following groups: first, the lower quartile of EPL distribution as a measure of a lax EPL state; second, the upper quartile of EPL distribution as a measure of a strict EPL state; and third, the rest of the observations (i.e., the interquartile range of the EPL distribution) as the measure of intermediate EPL. This kind of grouping allows us to identify differential effects across strict, intermediate, and lax EPL, where our interest lies mainly in looking at the difference between strict and lax EPL given that this gap reasonably captures a sufficiently large differentiation between EPL regimes for picking up any true effects in the data. While these policy regime dummies are time-varying, it is important to notice that the EPL index exhibits very small temporal variation, as opposed to relatively large cross-sectional variance, resulting in relative stability of policy regimes over long horizons.<sup>33</sup>

**Econometric Specification.** As in [Auerbach and Gorodnichenko \(2012\)](#), we make use of the [Jorda \(2005\)](#) local projections method within a fixed-effects panel model, where inference is based on [Driscoll and Kraay \(1998\)](#) standard errors that allow arbitrary correlations of the error term across countries and time. In particular, we estimate impulse responses to the credit supply shock by projecting a variable of interest on its own lags and contemporaneous and lagged values of the EBP variable from [Gilchrist and Zakrajšek \(2012\)](#), while allowing the estimates to vary according to the EPL state in a particular country and time.

The following equation demonstrates the class of state-dependent models that we es-

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<sup>33</sup>The relatively low temporal variance of EPL and labor market institutions is also noted in [Lazear \(1990\)](#) and [Gnocchi et al. \(2015\)](#).



timate using  $y$  as an example of a dependent variable.<sup>34</sup>

$$\begin{aligned}
\ln y_{i,t+h} - \ln y_{i,t-1} = & A_{i,t-4}[\alpha_{A,i}^h + \beta_A^h EBP_t + \Theta_A^h(L)EBP_{t-1} + \Gamma_A^h(L)\Delta \ln y_{i,t-1}] \\
& + B_{i,t-4}[\alpha_{B,i}^h + \beta_B^h EBP_t + \Theta_B^h(L)EBP_{t-1} + \Gamma_B^h(L)\Delta \ln y_{i,t-1}] \quad (32) \\
& + C_{i,t-4}[\alpha_{C,i}^h + \beta_C^h EBP_t + \Theta_C^h(L)EBP_{t-1} + \Gamma_C^h(L)\Delta \ln y_{i,t-1}] + \epsilon_{i,t+h}^h,
\end{aligned}$$

where  $i$  and  $t$  index countries and time;  $\alpha_i$  is the country fixed effect;  $\Theta(L)$  and  $\Gamma(L)$  are lag polynomials;  $\beta^h$  gives the response of the outcome variable at horizon  $h$  to a credit supply shock at time  $t$ ;  $\epsilon_{i,t+h}^h$  is the residual; and, importantly, all the coefficients vary according to the state of EPL which is represented by the state dummies  $A_{i,t-4}$ ,  $B_{i,t-4}$ , and  $C_{i,t-4}$  that take the value of one when the EPL regime is lax, intermediate, or strict as we defined above. The estimated impulse responses to the credit supply shock for the three states at horizon  $h$  are simply  $\beta_A^h$ ,  $\beta_B^h$ , and  $\beta_C^h$ , respectively.

Lags of  $y$  and EBP are included in the regression to remove any predictable movements in EBP; this facilitates the identification of an unanticipated shock to EBP, which is what is sought after. We assign the value of the order of lag polynomials  $\Theta(L)$  and  $\Gamma(L)$  to 8, i.e., we allow for 8 lags of the log-first-difference of the outcome variable and EBP in the regression. We assume a relatively large number of lags because of the construction of the EPL variable. Since the latter was converted from annual to quarterly frequency by assuming identical values within the year, it is necessary to include it in the regression with four lags to avoid correlation of the error term with it; this in turn requires that more than 4 lags of output and EBP be included in the regression to purge the state dummies of any potentially endogenous sources.<sup>35</sup>

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<sup>34</sup>In order to correctly adopt a state-dependent model for panel data, we must refer to a form of normalized changes in variables for these changes to be commensurable between countries. To accomplish such normalization, we simply use a dependent variable of the form  $\ln y_{i,t+h} - \ln y_{i,t-1}$  which represents the log-cumulative-difference in our variable of interest from the pre-shock horizon until horizon  $h$ .

<sup>35</sup>When using other data frequencies, we use two years of lagged values, following the same argument.

The EBP credit supply shock is normalized so that it has a zero mean and unit variance. Note that a separate regression is estimated for each horizon. We estimate a total of 21 regressions for our quarterly frequency specification and collect the impulse responses from each estimated regression, allowing for an examination of the state-dependent effects of credit supply shocks for 5 years following the shock.

Our form of state-dependence is slightly different from the conventional one (see, e.g., [Ramey and Zubairy \(2017\)](#)) which usually uses a binary state variable. Our identification utilizes an ordered ranking system by breaking down the raw EPL measure into 3 different ordered EPL regimes. If EPL's strictness indeed causes a change in the response of a certain variable then we would expect to see that its responses to the shock across EPL regimes will maintain an ordered pattern, i.e.,  $\beta_A^h > \beta_B^h > \beta_C^h$  or  $\beta_A^h < \beta_B^h < \beta_C^h$ . Note that our identification does not assume anything that would guarantee such an ordering unless it is present in the data, unlike the results that would have been obtained from a continuous interaction exercise. In Online Appendix B we conduct an analysis of the results' robustness to the choice of cutoff values for the policy regime dummies to ensure that our results are not driven by our baseline cutoff value choices.

## 6 Empirical Analysis

In this section, we perform an empirical analysis of EPL's implications for economic resilience, utilizing the aforementioned identification method. Section 6.1 presents our results regarding the shock's effects focusing on the differential response patterns which are conditional upon the EPL regime in place. The subsequent Section 6.2 presents a more in-depth examination of the causes behind the differential response patterns and their link to the labor-misallocation channel discussed in Section 3.

## 6.1 EPL's Implications for Business Cycle Dynamics

We estimate the state-dependent specification described in Equation (32) for output, consumption, investment, government expenditure, imports, exports, the real wage, the stock of vacancies, employment to population ratio, labor-force participation, and unemployment. The estimation results are presented in Figures 4 and 5, where the responses of economies under a strict EPL regime are shown in blue, for those under a lax regime in red, and those in the intermediate regime in black.

Regardless of the EPL state, the credit supply shock causes the expected dynamics, i.e., an increase in unemployment and a decrease in real activity measures (most importantly, a decrease in real output, consumption and investment). Our interest lies in the differences of responses across the policy regimes, whose statistical significance is indicated by the shaded areas in Figures 4 and 5.

**Labor Market Outcomes.** The first form of differential response to arise between the policy regimes is in the labor market and it is presented in Figure 4. Being in a lax EPL state produces an immediate increase in unemployment and a decrease in employment while being in a strict EPL state generates no significant change in unemployment until a year after the shock and no statistically significant decrease in employment at all horizons. This is in line with the slower turnover suggested by the model simulation from Section 3. This pattern also agrees with the notion that job-destruction is less counter-cyclical under a strict EPL regime, thus making overall employment less responsive.<sup>36</sup> Notably, the labor market in the lax EPL state manages to recover back to steady state significantly faster than in the strict EPL state, with the unemployment rate and vacancies responses during the later phase of the cycle being significantly higher and lower, respectively, in the strict EPL state relative to the lax one.

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<sup>36</sup>See Bentolila and Bertola (1990), Garibaldi (1998), and Nunziata (2003).

A difference observed across EPL states from which we abstract in our theoretical analysis is that labor-force participation is adversely affected by the shock in the lax EPL state while being in the strict EPL state produces no such effect. The effect on participation could be interpreted from a structural standpoint as being driven by the relatively higher value of the job-seeker from a future match with an employer, anticipating a longer employment duration which lowers discouragement from costly search activities.

**National Accounts.** The second form of differential response is the response of real activity measures presented in Figure 5. One year after the shock, we begin to see that real output starts to decline more in the strict EPL state than in the lax EPL state. This gap in output is steadily widening, starting to be significantly different from zero from the 7th quarter onwards and translating to a relative cumulative output loss of 0.75% after 2 years, 1.31% after three years, 2.18% after four years, and a peak 2.40% after five years.<sup>37</sup> In Online Appendix B, we show that this response pattern is robust to cutoff values' selection, lag order selection, and alternative sample and output measure choices.

Other measures of real activity do not exhibit any statistically significant differential response pattern until at least two years after the shock. Consumption starts to decline in a significantly differential fashion from the 9th quarter onwards. For investment, a significant differential decline occurs from the 11th quarter onwards. Imports fall differentially from the 10th quarter onwards quarters whereas exports begin to decline differentially after 5 quarters, but only until the 7th quarter and then again after 12 quarters up to the 15th quarter (and at somewhat lower confidence levels relative to the other variables, with p-values always exceeding 5%). These differential responses all occur in the same direction as that of output's response, i.e., being in a strict EPL state generates a stronger decline

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<sup>37</sup>We present all our results for a five-year horizon. However, to test that this effect does not grow further in magnitude, we estimated the corresponding difference after six years to be 1.55% using the same methods explained above.

in all these real activity measures relative to being in the lax EPL state. It is noteworthy that these differential responses all occur in the absence of any persistent significant changes in the real wage in all EPL regimes with similarly weak responses of government expenditures.

Linking the results from Figure 5 to those from Figure 4, it is important to observe that the initially stronger decline in employment from the latter figure occurs under the lax policy regime while the following stronger drop in output occurs under the strict one, with no differential response in employment taking place after the first two years. Moreover, the differential response of investment would not be able to account for any significant diminution in the capital stock available for production until at least three years after the shock (i.e., the decline in output precedes the drop in capital stock and not vice versa), and even then the differences are not strong enough to explain the differential output response by themselves.<sup>38</sup> In other words, the difference in output response across the policy regimes is too strong to be explained solely by changes in factor inputs at any point in time, giving rise to what at first pass seems like a contradiction.

However, viewing the results from Figures 5 and 4 through the lens of our model points towards the likely presence of labor-misallocation-induced TFP decline in the strict policy regime. If we recall Eq. (23), the change in output in our model consisted of four elements: pure technology, capital, labor, and labor misallocation. As argued before, especially for the first few years after the shock, aggregate capital stock is cannot account for any differential effect across the policy regimes. Pure aggregate technological level is unlikely to be affected differentially as a result of a common shock. We are left, therefore, with aggregate labor and labor-misallocation. For the first year and a half after the shock,

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<sup>38</sup>To illustrate, if we were to assume a 10% annual depreciation rate of the capital stock, and use the exact cumulative changes in investment from Figure 5, assuming that both EPL groups begin from the same level of steady-state capital stock, the differences between the capital stock in the strict and lax policy regimes will be less than 0.1% for the first three years of the cycle, 0.54% for the fourth year and 1.07% for the fifth one.

employment decline significantly more in the lax regime than in the strict one. As such, one would expect that output would decline by more in the lax group. However, the responses of output are remarkably close to one another. As such, there should be another force that pushes output down from the onset of the cycle. In the next section, we would argue that this force consists of, at least partially, from the labor misallocation channel we have discussed in the previous sections that would drive TFP down in the strict EPL economies.

## 6.2 The Differential Output Drop.

To analyze the differential output drop and understand its causes we proceed by asking the following questions: First, are we accurately accounting for the labor input actually used in production? And second, is TFP indeed responding in a fashion that could explain the differential responses in output across EPL states?

To account for better measurement of labor input, we use data on actual hours worked. Notwithstanding the lower, annual frequency of this series, using it has the potential of better measuring true variation in input quantity than using the number of employed persons. Next, if we consider a generalized production function then the real output will be determined by raw inputs' quantities, the degree to which they are utilized, and the level of TFP. With these two considerations in mind, we estimate the impulse responses of total hours worked and TFP, at an annual frequency, as well as those of capacity utilization at a quarterly frequency, again conditioning on the initial regime of EPL in place using the same identification as before.<sup>39</sup> Capacity utilization has the potential to confound our conclusions regarding TFP since our measure of TFP is not utilization-adjusted. It is therefore important to also look at the behavior of utilization as jointly examining un-

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<sup>39</sup>Detailed description of the data series used can be found in Online Appendix C.

adjusted TFP and utilization can paint a much clearer picture regarding the behavior of utilization-adjusted TFP that includes misallocation and technology level.

The results of this exercise are shown in Figure 6. There is a significantly smaller initial decline in hours worked in the lax EPL state which reverses in a significant way in the fourth and fifth years after the shock, with hours worked returning significantly faster to steady state during these years in the lax EPL state. This response pattern is in line with our previous measure of labor input and supports the conclusion that employment as an input in production responds less when EPL is strict.

TFP declines in the strict EPL state while under a lax regime TFP is not affected by the shock in a statistically significant way. This difference in TFP responses is sizable, peaking at 0.77% after three years, and statistically significant from the first through the third year. The forms of the impulse responses are quite similar to those obtained from our simulation exercise in Section 2. So why does output take longer than TFP to respond differentially? Due to the conflicting effects of the slower change in employment and the stronger response of TFP on aggregate output, it is only when the difference in employment subsides that the TFP difference has a chance to manifest into a differential output drop. In terms of timing, this only holds after the first year of the cycle. What about the recovery period?

Capacity utilization, which can be thought of as a proxy for factor utilization, behaves in a significantly different manner that can at least in part also account for the differential output response during the later phase of the cycle. Overall, across all 3 EPL states, we see that the beginning of the cycle is associated with a decrease in capacity utilization. However, the persistence of the decline in utilization is varying according to the initial state of EPL. For the first three years, during which the above-mentioned TFP channel is present, there are no differences in responses of utilization after the second quarter following the credit supply shock. After 10 quarters we begin to see a diverging pattern

of recovery that is significant from about 3.5 years onwards, with utilization recovering much faster in the lax EPL state and the associated response difference peaking at 1.71% after 17 quarters. These results, which are also in accordance with the differential recovery of hours worked during the same time frame, indicate that the differential behavior of utilization in the later stage of the cycle constitutes an important contribution to the correspondingly stronger output drop in the strict EPL state relative to the lax one. The fact that capacity utilization does not respond in a differential manner for the period of 1 year to 3 years following the shock, brings us back to the misallocation channel.

**EPL and Misallocation: An Empirical Perspective.** The differential response pattern of utilization does not occur during the same time as that of TFP, suggesting that the observed effect on TFP across the policy regimes does not stem from differences in utilization. Hence, the results from Figure 6 indicate that the stronger drop in TFP is likely driven by non-utilization-related forces which in turn strengthen the effect of the original shock on aggregate output. This amplification mechanism further enhances the cycle's strength, contributes to its persistence, and leads to a slower recovery of the economy as a whole. Importantly, since our TFP measure is unadjusted for factor utilization changes and the differential drop in utilization takes place only after that in TFP occurs, we infer from the empirical evidence that a potentially important channel underlying TFP's differential decline lies in increased factor misallocation taking place in the strict EPL state.<sup>40</sup> Specifically, our results indicate that the stronger output decline in the first 3 years after the shock can be explained by a factor-misallocation-induced TFP decline, whereas the subsequent two-year differential output fall seems to be mostly driven by a correspond-

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<sup>40</sup>Underlying this factor misallocation based interpretation is the assumption that technology is unaffected by credit supply shocks, which is what the literature on the TFP channel of credit supply shocks normally assumes (see, e.g., Buera et al. (2011), Pratap and Urrutia (2012), Petrosky-Nadeau (2013), Khan and Thomas (2013), Buera and Moll (2015), Buera et al. (2015), Gopinath et al. (2017), Buera and Shin (2017), and Manaresi and Pierri (2017)).



ing differential drop in factor utilization and hours worked.

At the core of this interpretation of the results is the assertion that the joint prevalence of the adverse shock and strict EPL is associated with more misallocation. The persistence of this factor misallocation stems from a slower reallocation of labor. In addition to it being an important channel in our model, this assertion receives both theoretical and empirical support in the literature on EPL. A theoretical example of this connection can be found in the work of [Garibaldi \(1998\)](#) which concludes from a stochastic search model that firing restrictions reduce labor reallocation and slow turnover. More recent empirical evidence linking EPL and factor misallocation, in general, lend support this claim. [Caballero et al. \(2013\)](#) find that stricter EPL, especially with respect to dismissal regulations, is linked to a lower speed of adjustment to shocks which in turn lowers productivity growth, a process which they connect to Schumpeter's idea of 'creative destruction'. Using a difference in differences estimation and industry-level data, [Bassanini et al. \(2009\)](#) show that EPL strictness is associated with a lower productivity growth rate and that this effect is due to the binding limitation on termination which may lead to a lower change in aggregate productivity unless the market is extremely centered around industries for which terminations are not the primary source of turnover. [Petrin and Sivadasan \(2013\)](#) find from plant-level evidence in Chile's manufacturing industry that there is a reason to believe that changes in severance pay are responsible for an increase in the gap between the value of the employees' marginal product and their wage. This gap measures, in fact, the allocation inefficiency, which means that the introduction of stricter termination regulations in Chile may have induced an increase in factor misallocation. The work of [Lashitew \(2016\)](#) provides further support to this claim by using plant-level data to show that there is a link between EPL strictness and factor-misallocation-induced productivity losses.

**Comparison to the Model Predictions - Attempting to Gauge the Quantitative Importance of the Labor Misallocation Channel.** The quantitative results in our baseline calibration can account for a 0.139% TFP response to a 10% increase in the net cost of capital, and up to 0.209% in the HM-style calibration. These are the peak effects and are obtained in the first year after the shock. The empirical results indicate that the differential drop in TFP is about 0.445% after one year. One standard error of EBP during 1985-2013 corresponds to 0.567 percentage points where the average of the [Gilchrist and Zakrajšek \(2012\)](#) credit spread (GZ spread) is 2.082 percentage points. If one considers the risk-free rate as 4 percent which is the accepted value for calibration in DSGE models, one would obtain that the average capital price is about 6 percent and that a one standard error shock in EBP is approximately a 10% increase in the net price of capital. Thus, these two sets of impulse responses can be treated as if they are in response to a shock of roughly the same magnitude.

The channel of amplification we present in our theory has as its ultimate result a differential output drop. This drop is a salient and robust feature of our empirical analysis as we will demonstrate in Online Appendix B. What is the source of this drop? The difference in TFP responses causes output to be hit more severely in the countries with stricter firing restrictions. In our model, the TFP drop has its roots in changes in the cross-sectional misallocation of labor that originates from increased separations. Another robust feature of our results is that unemployment rises in response to the shock. If this rise in unemployment is entirely driven by a decline in hiring, we would have no cause to conclude that this is our misallocation channel. However, if this increase in unemployment is in part, or in full, due to a rise in separations, this would support our conclusion that the TFP decline is driven by an increase in labor misallocation which originates from firing restrictions. Note that separations do not need to respond differently across policy regimes to elicit a differential TFP response. In the perfectly flexible case with no firing

restrictions,  $l = 0$  and  $\phi \rightarrow \infty$ , the amplification parameter  $\psi$  in Eq. (31) goes to zero. Thus, even given the same increase in separations, the strict policy group would exhibit a larger TFP drop and a stronger response in terms of real output.

The first piece of evidence that supports the existence of an increase in separations in the data is laid out in Figure 4. Under the strict EPL regime, vacancies do not exhibit any statistically significant decline during the first two years after the shock. However, data on vacancy stock is available for only a partial sample of countries and periods. Thus, although increased separations in times of recession is a likely scenario we cannot conclude that it is so based upon this dataset alone. With this in mind, we turn our attention to a more detailed analysis of job flows.

**Evidence From Job Flows.** Since job flow data is not as readily available as employment and unemployment data, we use decomposed flow hazards from the work of [Elsby et al. \(2013\)](#) to examine if it is indeed the case that separations rise in response to our shock. The authors combine OECD data and additional surveys to compile data-series of job-finding rates and separation rates at an annual frequency for 14 countries in our sample for varying time frames until 2009.<sup>41</sup> We use this data to examine the response of job flows to our shock. Results from this exercise are summarized in Figure 7.

Without regard to the policy regime in question, the response of the logged hazard rates to the shock is in line with our previous results. The job-finding rate decreases in response to the shock and the separation rate increases albeit these responses are mostly not statistically significant other than at several particular horizons. When looking at the impulse responses given in Figure 7 it is important to bear in mind that the aftermath of the strongest realizations of our shock (i.e., those taking place in 2008) is mostly absent

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<sup>41</sup>For additional details see Online Appendix C. The reason we do not use the aforementioned [Hobijn and Sahin \(2009\)](#) data as in the calibration, although it is available for more countries, is that these correspond to the steady-state values rather than the time-series.

from the sample and that the data is of low frequency. Hence, our identification of credit supply shocks is likely undermined in this exercise with the associated analysis probably suffering from a lack of statistical power. However, it suggests several very likely conclusions at the horizons where we have statistical significance and in terms of overall dynamics.

First, note that the response of the job-finding rate across the policy regimes is very much in line with the results regarding vacancies. Namely, job-finding decreases in a statistically significant fashion in the lax EPL group but not in the strict group. This supports the conclusion that the increase in unemployment is due, at least in part, to an increase in separations. Second, in the strict EPL group, separations increase in a statistically significant way and rather strongly during the third year after the shock with an increase of 10%. A smaller and not statistically significant effect is present at the first two years after the shock with a more moderate increase of about 3% in the separation rate. Taken together with our previous results, this set of results lead us to conclude that the rise in unemployment originates, at least partly, from a rise in separations that would generate the required cross-sectional misallocation we model in Section 2.

The responses of the job flows in the data are very much in line with what we would expect from the model's broad prediction for a contractionary business cycle, i.e., job-finding goes down and separation goes up. Our results also suggest that the labor-misallocation channel which amplifies the decline in TFP and output is present. We do not claim that this is the only channel of influence on TFP in response to the shock, only that it is present and is a significant one as our model prediction is between 31% to 47% of the observed effect on output.

However, the exact quantification of this channel is somewhat challenging. It is conceptually possible to compute and compare values of  $\psi$  for each country and quantify in the data the effect of the labor misallocation channel from the total TFP decline. However,

this is a very difficult problem in term of data for several reasons. First, one would have to have a measure of the costs of termination across countries which requires micro-data of very high quality; an example of an exercise of this type for France one can find in [Kramarz and Michaud \(2010\)](#). Second, termination notice duration varies within many countries by industry and or occupation via collective agreements. Whenever one encounters the legislated notice period, it should be treated as a legal minimum. Higher notice periods can be bargained for collectively or individually. Third, these should be combined with industry levels of the labor share and average job flows. These challenges place an exercise of this nature beyond the scope of our current work.

To recap, the strictness of EPL slows job flows and contributes to misallocation in the presence of the adverse shock. In other words, combined with the adverse shock, strict EPL enhances the sullyng effect of the business cycle and lowers TFP via misallocation. This transmission channel makes the economy less resilient overall to shocks as it amplifies aggregate output response and leads to a slower recovery.

## 7 Conclusion

This paper demonstrates the potential of firing restrictions to amplify macroeconomic shocks in three steps. First, using a stylized search and matching model, we illustrate the contribution of firing costs and termination notice practices to the propagation of shocks via misallocation-induced TFP decline. Second, using a quantitative version of our model we show how a stochastic shock to the cost of capital causes a more pronounced output drop in an economy with firing restriction and that TFP decline is at the source of this variation. Last, we perform an empirical analysis of a panel of 21 countries aimed at examining the relationship between firing restriction and economic resilience using a state-dependent local projections based identification strategy.

Our findings indicate that firing restrictions' strictness has the capacity to act as an amplifier of macroeconomic shocks. While diminishing the decrease in employment following an adverse credit supply shock, it severely hinders the recovery of real output to pre-shock levels. This sizable and robust relative decline in real activity seems to originate from a TFP decline that is present in the strict EPL state and is absent in the lax EPL one. This paper argues that the mechanism underlying this relative decline in TFP is derived from the misallocation present in our model and in our simulation exercise.

Our results shed light on an often-overlooked element in the policy debate which surrounds labor market policies, which is their effects on business cycle dynamics. The results are especially relevant for the current COVID-19 crisis, during which labor market turnover is at an unprecedented level. Our results indicate that in those countries that have high levels of firing restrictions, relaxing these measures during the crisis may allow for a faster recovery of real output. This recommendation, alongside other measures which are currently a part of the lively policy debate, may prove useful for the design of a recovery strategy in many advanced economies.

This paper points towards several directions for future work. From a theoretical standpoint, this paper may prove of value for model builders in the construction of models that can accommodate the type of link between EPL, TFP, and varying factors' utilization observed in the data conditional on a shock-induced business cycle. From an empirical standpoint, our model suggests a new way to account for misallocation of labor in TFP measurement via our aggregation results. The capacity to consider these channels of influence may prove useful in producing cleaner measures of aggregate TFP and improve our understanding of aggregate fluctuations.

## References

- Arnold, B. C.: 2008, *Pareto and Generalized Pareto Distributions*, Springer New York, New York, NY, pp. 119–145.
- Auerbach, A. J. and Gorodnichenko, Y.: 2012, Measuring the output responses to fiscal policy, *American Economic Journal: Economic Policy* **4**(2), 1–27.
- Barlevy, G.: 2002, The Sullyng Effect of Recessions, *The Review of Economic Studies* **69**(1), 65–96.
- Bassanini, A., Nunziata, L. and Venn, D.: 2009, Job protection legislation and productivity growth in oecd countries, *Economic Policy* **24**, 349–402.
- Basu, S., Fernald, J. G. and Kimball, M. S.: 2006, Are technology improvements contractionary?, *American Economic Review* **96**(5), 1418–1448.
- Bentolila, S. and Bertola, G.: 1990, Firing costs and labour demand: How bad is euroclerosis?, *The Review of Economic Studies* **57**(3), 381–402.
- Bentolila, S., Cahuc, P., Dolado, J. J. and Le Barbanchon, T.: 2012, Two-tier labour markets in the great recession: France versus spain\*, *The Economic Journal* **122**(562), F155–F187.
- Bentolila, S. and Saint-Paul, G.: 1992, The macroeconomic impact of flexible labor contracts, with an application to spain, *European Economic Review* **36**(5), 1013–1047.
- Blanchard, O. and Wolfers, J.: 2000, The role of shocks and institutions in the rise of european unemployment: the aggregate evidence, *The Economic Journal* **110**(462), 1–33.
- Boeri, T. and Garibaldi, P.: 2007, Two tier reforms of employment protection: a honeymoon effect?, *Economic Journal* **117**(521), 357–385.

- Boeri, T. and van Ours, J.: 2013, *The Economics of Imperfect Labor Markets: Second Edition*, stu - student edition edn, Princeton University Press.
- Buera, F. J., Jaef, R. N. F. and Shin, Y.: 2015, Anatomy of a credit crunch: From capital to labor markets, *Review of Economic Dynamics* **18**(1), 101 – 117.
- Buera, F. J., Kaboski, J. P. and Shin, Y.: 2011, Finance and development: A tale of two sectors, *American Economic Review* **101**(5), 1964–2002.
- Buera, F. J. and Moll, B.: 2015, Aggregate implications of a credit crunch: The importance of heterogeneity, *American Economic Journal: Macroeconomics* **7**(3), 1–42.
- Buera, F. J. and Shin, Y.: 2017, Productivity growth and capital flows: The dynamics of reforms, *American Economic Journal: Macroeconomics* **9**(3), 147–85.
- Caballero, R. J., Cowan, K. N., Engel, E. M. and Micco, A.: 2013, Effective labor regulation and microeconomic flexibility, *Journal of Development Economics* **101**(C), 92–104.
- Cahuc, P., Charlot, O. and Malherbet, F.: 2016, Explaining the spread of temporary jobs and its impact on labor turnover, *International Economic Review* **57**(2), 533–572.
- den Haan, W. J., Ramey, G. and Watson, J.: 2000, Job destruction and propagation of shocks, *American Economic Review* **90**(3), 482–498.
- Driscoll, J. C. and Kraay, A. C.: 1998, Consistent covariance matrix estimation with spatially dependent panel data, *Review of economics and statistics* **80**(4), 549–560.
- Duval, R. and Vogel, L.: 2008, Economic resilience to shocks, **2008**.
- Elsby, M. W. L., Hobijn, B. and Şahin, A.: 2013, Unemployment Dynamics in the OECD, *The Review of Economics and Statistics* **95**(2), 530–548.



- EUR-Lex Directive 98/59.: 2016, *Technical report*. Available at: [Accessed 13 Apr. 2016].
- Fisher, J. D.: 2015, On the Structural Interpretation of the Smets–Wouters “Risk Premium” Shock, *Journal of Money, Credit and Banking* **47**(2-3), 511–516.
- Gaetani, R. and Doepke, M.: 2016, Employment Protection, Investment in Job-Specific Skills, and Inequality Trends in the United States and Europe, *Technical report*.
- Garibaldi, P.: 1998, Job flow dynamics and firing restrictions, *European Economic Review* **42**(2), 245–275.
- Gilchrist, S. and Zakrajšek, E.: 2012, Credit spreads and business cycle fluctuations, *American Economic Review* **102**(4), 1692–1720.
- Gnocchi, S., Lagerborg, A. and Pappa, E.: 2015, Do labor market institutions matter for business cycles?, *Journal of Economic Dynamics and Control* **51**(C), 299–317.
- Gopinath, G., Kalemli-Özcan, Ş., Karabarbounis, L. and Villegas-Sanchez, C.: 2017, Capital allocation and productivity in south europe, *The Quarterly Journal of Economics* **132**(4), 1915–1967.
- Hagedorn, M. and Manovskii, I.: 2008, The cyclical behavior of equilibrium unemployment and vacancies revisited, *American Economic Review* **98**(4), 1692–1706.
- Hall, R. E.: 2017, High discounts and high unemployment, *American Economic Review* **107**(2), 305–30.
- Hall, R. E. and Milgrom, P. R.: 2008, The limited influence of unemployment on the wage bargain, *American Economic Review* **98**(4), 1653–74.
- Hobijn, B. and Sahin, A.: 2009, Job-finding and separation rates in the oecd, *Economics Letters* **104**(3), 107–111.

- Hornstein, A., Krusell, P. and Violante, G. L.: 2011, Frictional wage dispersion in search models: A quantitative assessment, *American Economic Review* **101**(7), 2873–98.
- Hsieh, C.-T. and Klenow, P.: 2009, Misallocation and manufacturing tfp in china and india, *The Quarterly Journal of Economics* **124**(4), 1403–1448.
- Jorda, O.: 2005, Estimation and inference of impulse responses by local projections, *American Economic Review* **95**(1), 161–182.
- Kahn, L. M.: 2007, The impact of employment protection mandates on demographic temporary employment patterns: International microeconomic evidence, *The Economic Journal* **117**(521), F333–F356.
- Khan, A. and Thomas, J. K.: 2013, Credit shocks and aggregate fluctuations in an economy with production heterogeneity, *Journal of Political Economy* **121**(6), 1055–1107.
- Kramarz, F. and Michaud, M.-L.: 2010, The shape of hiring and separation costs in france, *Labour Economics* **17**(1), 27–37.
- Lagos, R.: 2006, A model of tfp, *The Review of Economic Studies* **73**(4), 983–1007.
- Lashitew, A. A.: 2016, Employment protection and misallocation of resources across plants: International evidence, *CESifo Economic Studies* **62**(3), 453–490.
- Lazear, E. P.: 1990, Job security provisions and employment, *The Quarterly Journal of Economics* **105**(3), 699–726.
- Ljungqvist, L. and Sargent, T. J.: 2017, The fundamental surplus, *American Economic Review* **107**(9), 2630–65.
- Manaresi, F. and Pierri, N.: 2017, Credit supply and productivity growth, *Working paper*, Stanford University.

- Messina, J. and Vallanti, G.: 2007, Job flow dynamics and firing restrictions: Evidence from europe, *The Economic Journal* **117**(521), F279–F301.
- Moll, B.: 2014, Productivity losses from financial frictions: Can self-financing undo capital misallocation?, *American Economic Review* **104**(10), 3186–3221.
- Myant, M. and Brandhuber, L.: 2016, Uses and abuses of the oecd’s employment protection legislation index in research and eu policy making, *Working paper*, ETUI.
- Nickell, W.: 2006, The cep-oecd institutions data set (1960-2004), *Lse research online documents on economics*, London School of Economics and Political Science, LSE Library.
- Nunziata, L.: 2003, Labour market institutions and the cyclical dynamics of employment, *Labour Economics* **10**(1), 31 – 53.
- Nunziata, L. and Staffolani, S.: 2007, Short-term contracts regulations and dynamic labour demand: Theory and evidence, *Scottish Journal of Political Economy* **54**(1), 72–104.
- Ohanian, L. E.: 2010, The economic crisis from a neoclassical perspective, *Journal of Economic Perspectives* **24**(4), 45–66.
- Ohanian, L. E. and Raffo, A.: 2012, Aggregate hours worked in OECD countries: New measurement and implications for business cycles, *Journal of Monetary Economics* **59**(1), 40–56.
- Owyang, M. T., Ramey, V. A. and Zubairy, S.: 2013, Are government spending multipliers greater during periods of slack? evidence from 20th century historical data, *American Economic Review, Papers and Proceedings* **103**(3), 129–134.
- Perri, F. and Quadrini, V.: 2018, International recessions, *American Economic Review* **108**(4-5), 935–84.

- Petrin, A. and Sivadasan, J.: 2013, Estimating lost output from allocative inefficiency, with an application to chile and firing costs, *The Review of Economics and Statistics* **95**(1), 286–301.
- Petrosky-Nadeau, N.: 2013, TFP during a credit crunch, *Journal of Economic Theory* **148**(3), 1150 – 1178.
- Pissarides, C. A.: 2000, *Equilibrium Unemployment Theory, 2nd Edition*, Vol. 1 of MIT Press Books, The MIT Press.
- Pratap, S. and Urrutia, C.: 2012, Financial frictions and total factor productivity: Accounting for the real effects of financial crises, *Review of Economic Dynamics* **15**(3), 336 – 358.
- Ramey, V. A. and Zubairy, S.: 2017, Government spending multipliers in good times and in bad: Evidence from u.s. historical data, *Journal of Political Economy* (forthcoming) .
- Restuccia, D. and Rogerson, R.: 2008, Policy Distortions and Aggregate Productivity with Heterogeneous Plants, *Review of Economic Dynamics* **11**(4), 707–720.
- Rumler, F. and Scharler, J.: 2011, Labor market institutions and macroeconomic volatility in a panel of oecd countries, *Scottish Journal of Political Economy* **58**(3), 396–413.
- Saint-Paul, G.: 2000, *The Political Economy of Labour Market Institutions*, Oxford University Press.
- Saint-Paul, G.: 2002, Employment protection, international specialization, and innovation, *European Economic Review* **46**(2), 375 – 395.
- Shimer, R.: 2005, The cyclical behavior of equilibrium unemployment and vacancies, *American Economic Review* **95**(1), 25–49.
- Skedinger, P.: 2010, *Employment Protection Legislation*, Edward Elgar Publishing.

Tenreyro, S. and Thwaites, G.: 2016, Pushing on a string: Us monetary policy is less powerful in recessions, *American Economic Journal: Macroeconomics* 8(4), 43–74.

Zanetti, F.: 2011, Labor market institutions and aggregate fluctuations in a search and matching model, *European Economic Review* 55(5), 644–658.

## Appendix A Model Solution

### A.1 Deriving The Match Surplus Equation

Recall the definition of the surplus from Eq. (6):

$$M(x, \mathbf{s}) = J(x, \mathbf{s}) + W(x, \mathbf{s}) - M_n(\mathbf{s}).$$

Multiplication by  $r$  and substituting in Eq. (1) and (3) yields:

$$\begin{aligned} rM(x, \mathbf{s}) &= w(x, \mathbf{s}) + \lambda \int_{x_{\min}}^{x_{\max}} \max \{ W(y, \mathbf{s}), W^n(w(x, \mathbf{s}), \mathbf{s}) \} dG(y) - \lambda W(x, \mathbf{s}) \\ &\quad + \tau E[\max \{ W(x, \mathbf{s}'), W^n(w(x, \mathbf{s}), \mathbf{s}') \} - W(x, \mathbf{s}) \mid \mathbf{s}] + xp[f(k(\mathbf{s})) - \rho k(\mathbf{s})] - w(x, \mathbf{s}) \\ &\quad + \lambda \int_{x_{\min}}^{x_{\max}} \max \{ J(y, \mathbf{s}), J^n(w(x, \mathbf{s}), \mathbf{s}) \} dG(y) - \lambda J(x, \mathbf{s}) \\ &\quad + \tau E[\max \{ J(x, \mathbf{s}), J^n(w(x, \mathbf{s}), \mathbf{s}') \} - J(x, \mathbf{s}) \mid \mathbf{s}] - rM_n(\mathbf{s}). \end{aligned}$$

Canceling out the wage, using the definitions of  $M(x, \mathbf{s})$  and  $M_n(\mathbf{s})$ , and the identity

$M(x, \mathbf{s}) + M_n(\mathbf{s}) = J(x, \mathbf{s}) + W(x, \mathbf{s})$  one obtains:

$$\begin{aligned} (r + \lambda + \tau)(M(x, \mathbf{s}) + M_n(\mathbf{s})) &= xp(f(k(\mathbf{s})) - \rho k(\mathbf{s})) + \\ &\quad + \lambda \left[ \int_{x_{\min}}^{x_{\max}} \max \{ W(y, \mathbf{s}), W^n(w(x, \mathbf{s}), \mathbf{s}) \} dG(y) + \int_{x_{\min}}^{x_{\max}} \max \{ J(y, \mathbf{s}), J^n(w(x, \mathbf{s}), \mathbf{s}) \} dG(y) \right] \\ &\quad + \tau [E[\max \{ W(x, \mathbf{s}'), W^n(w(x, \mathbf{s}), \mathbf{s}') \} \mid \mathbf{s}] + E[\max \{ J(x, \mathbf{s}), J^n(w(x, \mathbf{s}), \mathbf{s}') \} \mid \mathbf{s}]]. \end{aligned}$$

This equation can be simplified further by keeping in mind that the first order conditions of the problem impose a surplus sharing of the form  $W(x, \mathbf{s}) - W^n(w(x, \mathbf{s}), \mathbf{s}) =$

$\beta M(x, \mathbf{s})$  and  $J(x, \mathbf{s}) - J^n(w(x, \mathbf{s}), \mathbf{s}) = (1 - \beta)M(x, \mathbf{s})$ . This means that  $W(x, \mathbf{s}) > W^n(w(x, \mathbf{s}), \mathbf{s})$  and  $J(x, \mathbf{s}) > J^n(w(x, \mathbf{s}), \mathbf{s})$  if and only if  $M(x, \mathbf{s}) > 0$ , which imply that the value of the expression inside all the maximum operators will be determined solely by  $M(x, \mathbf{s})$ . This results in

$$\begin{aligned} (r + \lambda + \tau)(M(x, \mathbf{s}) + M_n(\mathbf{s})) &= xp(f(k(\mathbf{s})) - \rho k(\mathbf{s})) \\ &+ \lambda \int_{x_{\min}}^{x_{\max}} \max(M(y, \mathbf{s}) + M_n(\mathbf{s}), M_n(\mathbf{s})) dG(y) \\ &+ \tau [E[\max\{M(x, \mathbf{s}') + M_n(\mathbf{s}'), M_n(\mathbf{s}')\} | \mathbf{s}]], \end{aligned}$$

which can be further simplified into:

$$\begin{aligned} (r + \lambda + \tau)(M(x, \mathbf{s}) + M_n(\mathbf{s})) &= xp(f(k(\mathbf{s})) - \rho k(\mathbf{s})) + \tag{A.1} \\ \lambda \left[ M_n(\mathbf{s}) + \int_{x_{\min}}^{x_{\max}} \max(M(y, \mathbf{s}), 0) dG(y) \right] &+ \tau [E[\max\{M(x, \mathbf{s}'), 0\} + M_n(\mathbf{s}') | \mathbf{s}]]. \end{aligned}$$

## A.2 Uniqueness of The Reservation Level

This section proves Lemma 2.2, which states that if there is any production at an aggregate state  $\mathbf{s}$ , then the match surplus has a unique zero, which we call the reservation level.

Recall that the match surplus is given by Eq. (6) as:

$$\begin{aligned} (r + \lambda + \tau)(M(x, \mathbf{s}) + M_n(\mathbf{s})) &= xp(f(k(\mathbf{s})) - \rho k(\mathbf{s})) + \\ \lambda \left[ M_n(\mathbf{s}) + \int_{x_{\min}}^{x_{\max}} \max(M(y, \mathbf{s}), 0) dG(y) \right] &+ \tau [E[\max\{M(x, \mathbf{s}'), 0\} + M_n(\mathbf{s}') | \mathbf{s}]]. \end{aligned}$$

Denote the aggregate state-space of the model by  $\Lambda$  with finite cardinality  $a$  and let transitions be governed by the Markov matrix  $\Pi$  with  $\pi_{i,j}$  denoting the transition probability from state  $i$  to state  $j$ . We define for each  $x$  the continuation states of  $x$  as the aggregate states in which  $M(x, \mathbf{s}) \geq 0$ , and denote this subset as  $\Lambda^c(x) \subseteq \Lambda$ . The derivative of  $M(x, \mathbf{s})$  with respect to  $x$  is thus:

$$(r + \lambda + \tau) \frac{\partial M(x, \mathbf{s})}{\partial x} = p(f(k(\mathbf{s})) - \rho k(\mathbf{s})) + \tau \sum_{\mathbf{s}' \in \Lambda^c} \pi_{\mathbf{s}, \mathbf{s}'} \frac{\partial M(x, \mathbf{s}')}{\partial x}. \quad (\text{A.2})$$

We denote by  $\Delta_x M(x, \mathbf{s})$ , the column vector of length  $a$  which contains all the derivatives  $\frac{\partial M(x, \mathbf{s})}{\partial x}$ . For this part only, we explicitly spell-out the state dependence of all the parameters and choice variables. Thus, the derivative of the match surplus in each state with respect to  $x$  is given by the system:

$$\mu \Delta_x M(x, \mathbf{s}) = \mathbf{p}(\mathbf{s}) (f(k(\mathbf{s})) - \rho k(\mathbf{s})) + \tau \Pi^c(x) \Delta_x M(x, \mathbf{s}), \quad (\text{A.3})$$

where  $\mu$  is an  $a$  by  $a$  diagonal matrix whose entries are  $\mu_{\mathbf{s}, \mathbf{s}} = r(\mathbf{s}) + \lambda(\mathbf{s}) + \tau$ ,  $\mathbf{p}(\mathbf{s}) = [p(f(k(\mathbf{s})) - \rho k(\mathbf{s}))]$  denotes a column vector of length  $a$ , and  $\Pi^c(x)$  is the Markov matrix  $\Pi$  after substituting all entries  $\pi_{i,j}$  such that  $j \notin \Lambda^c(x)$  with zeros.<sup>42</sup> The solution is given by:

$$\Delta_x M(x, \mathbf{s}) = (\mu - \tau \Pi^c(x))^{-1} \mathbf{p}(\mathbf{s}), \quad (\text{A.4})$$

To see that the solution exists, and is unique, we first examine the matrix

$$T = (\mu - \tau \Pi^c(x)).$$

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<sup>42</sup>We do not consider state dependence of  $\tau$  as any such design can be equivalently represented by changing the elements of  $\Pi$ .



The diagonal entries of  $T$  are either  $(r(\mathbf{s}) + \lambda(\mathbf{s}) + \tau) - \tau \pi_{\mathbf{s},\mathbf{s}}$  or  $(r(\mathbf{s}) + \lambda(\mathbf{s}) + \tau)$  which are strictly positive since  $r$  and  $\lambda$  are the discount rate and a Poisson arrival rate, and  $0 \leq \pi_{\mathbf{s},\mathbf{s}} \leq 1$ . The off-diagonal elements are either zero or  $-\tau \pi_{\mathbf{s},\mathbf{s}'}$ . Taken together, these make  $T$  a Z-matrix. Moreover, the matrix is semi-positive since there exists a vector, namely,  $\mathbf{i}$  the  $a$  length unit vector, such that  $T\mathbf{i} > 0$ , where  $>$  is the element-wise order. That makes  $T$  a non-singular M-matrix which has the property of being inverse positive.

Second, in addition to  $T^{-1}$  having only non-negative entries, the vector  $[p(f(k(\mathbf{s})) - \rho k(\mathbf{s}))]$ , is strictly positive as the capital is chosen optimally from  $f'(k(\mathbf{s})) = \rho$ . We thus have, from Euler's homogeneous function theorem that

$$f(k(\mathbf{s})) - \rho k(\mathbf{s}) = (1 - \alpha) f(k(\mathbf{s})) > 0.$$

Finally, we obtain that solution to Eq. (A.4) is the result of multiplication of a non-singular matrix with non-negative entries with a strictly positive vector which results in  $\Delta_x M(x, \mathbf{s})$  being strictly positive for all states. Thus, the match surplus is strictly increasing in  $x$  and if it has a zero in state  $\mathbf{s}$ , then this zero is necessarily unique.<sup>43</sup>

Key feature to note about the derivatives is that they do not depend on  $x$  other than via the matrix  $\Pi^c(x)$  which stems from the non-linearity of the maximum operator in Eq. (6). Thus, as long as the matrix  $\Pi^c(x)$  does not change, we have that the surplus is linear and increasing in  $x$ . Since there are  $a$  states, without loss of generality we can order them by their reservation levels as follows  $R(1) \geq R(2) \geq \dots \geq R(a)$ , and define  $a + 1$  intervals on the support between them, the first of these will be  $[R(1), x_{\max}]$ , followed by  $[R(2), R(1))$ , until  $[x_{\min}, R(a))$ . For each one of these intervals, the form of  $\Pi^c(x)$  is the same as the dependence upon  $x$  comes into play here only from the separation possibility encapsulated within the option value. Thus, the function  $M(x, \mathbf{s})$  is piece-

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<sup>43</sup>If there is no zero, but there is production, that would be equivalent to the statement  $R(\mathbf{s}) = x_{\min}$ . If there is no production, that is to say that  $R(\mathbf{s}) > x_{\max}$ .

wise linear and increasing, with points of discontinuity for the derivative situated at each of the reservation levels.

### A.3 The Wage Solution in the Deterministic Case

In order to solve for the wage in the deterministic case, we first look at the firm's value function and the surplus without aggregate risk:

$$(r + \lambda)(M(x) + M_n) = xp[f(k) - \rho k] + \lambda \left[ M_n + \int_R^{x_{\max}} M(x) G(y) dy \right], \quad (\text{A.5})$$

$$(r + \lambda)J(x) = xp[f(k) - \rho k] - w(x) + \lambda \left[ J^n(w(x)) + \int_R^{x_{\max}} J(y) - J^n(w(x)) dG(y) \right]. \quad (\text{A.6})$$

Substituting  $x = R$  into Eq. (A.6) yields:

$$(r + \lambda)J(R) = Rp[f(k) - \rho k] - w(R) + \lambda \left[ J^n(w(R)) + \int_R^{x_{\max}} J(y) - J^n(w(R)) dG(y) \right]. \quad (\text{A.7})$$

Recall the definition of the reservation level as  $M(R) = 0$  and the surplus sharing rule that is the FOC for (B1)  $J(x) - J^n = (1 - \beta)M(x)$ . From Lemma 2.1, we know that the problems (B1) and (B2) split the same surplus level, thus

$$\int_R^{x_{\max}} J(y) - J^n(w(x)) dG(y) = \int_R^{x_{\max}} J(y) - J^n(w(R)) dG(y) = \int_R^{x_{\max}} (1 - \beta)M(y) dG(y).$$

Using this one can subtract the two above equations to obtain:

$$(r + \lambda)(J(x) - J(R)) = (x - R)p[f(k) - \rho k] - (w(x) - w(R)) + \lambda[J^n(w(x)) - J^n(w(R))]. \quad (\text{A.8})$$

Repeating the same process for Eq. (A.5) by subtracting from the equation, itself with  $M(R) = 0$  we obtain

$$(r + \lambda)M(x) = (x - R)p(f(k) - \rho k). \quad (\text{A.9})$$

Now, substituting the surplus sharing rule, and the value of

$$J^n(w(x)) = -\frac{w(x) + \phi F p f(k) - x_{\min} p(f(k) - \rho k)}{r + \phi},$$

into Eq. (A.8) to obtain:

$$(r + \lambda)((1 - \beta)M(x)) = (x - R)p[f(k) - \rho k] - \left(\frac{\lambda}{r + \phi} + 1\right)(w(x) - w(R)).$$

This expression will be further simplified if we substitute in Eq. (A.9) and reverse the signs to obtain:

$$\beta((x - R)p(f(k) - \rho k)) = \left(\frac{\lambda + r + \phi}{r + \phi}\right)(w(x) - w(R)),$$

which after rearranging yields

$$w(x) = \frac{r + \phi}{r + \phi + \lambda} \beta((x - R)p(f(k) - \rho k)) + w(R). \quad (\text{A.10})$$

What about  $w(R)$ ? Since  $M(R) = 0$  from the surplus sharing rule we can say that  $J(R) = J'(w(R))$ . Using this in Eq. (A.7) along with the value of  $J'(w(R))$  yields

$$0 = Rp[f(k) - \rho k] - w(R) + \lambda \left[ \int_R^{x_{\max}} (1 - \beta) M(y) dG(y) \right] - r \frac{x_{\min} p(f(k) - \rho k) - w(R) - \phi F p f(k)}{r + \phi},$$

which after some rearrangement results in

$$w(R) = \frac{r + \phi}{\phi} \left[ Rp[f(k) - \rho k] + \lambda(1 - \beta) \int_R^{\infty} M(y) dG(y) \right] + \frac{r\phi F p f(k) - rx_{\min} p(f(k) - \rho k)}{\phi}. \quad (\text{A.11})$$

The wage function that results is at the reservation takes into account the notice period's duration and production in its duration, the production value at  $R$  and the option to enter into a period of notice from any wage level in the future.

Table 1: Model Calibration and Stochastic Steady-State Values.

	Baseline - France (1)	No firing costs (2)	No notice (3)	No firing restrictions (4)
Parameter values				
$p$	0.4500	0.4500	0.4500	0.4500
$c$	1.2953	1.2953	1.2953	1.2953
$\lambda$	0.1756	0.1756	0.1756	0.1756
$\beta$	0.5000	0.5000	0.5000	0.5000
$\phi$	0.7500	0.7500	62.7500	62.7500
$F$	0.6231	-	0.6231	-
$z$	0.8425	0.8425	0.8425	0.8425
$r$	0.0100	0.0100	0.0100	0.0100
$\delta$	0.0200	0.0200	0.0200	0.0200
$\alpha$	0.3300	0.3300	0.3300	0.3300
$G(x)$		$G(x) = 1 - \left(\frac{1}{x}\right)^{1.61}$		
$q(\theta)$		$0.2509\theta^{-0.5}$		
Model stochastic steady state				
$u$	0.1442	0.2865	0.3887	0.4324
$n$	0.0392	0.0613	0.0012	0.0012
$e$	0.8166	0.6523	0.6102	0.5664
$\bar{x}$	1.9118	2.4327	2.8395	3.1014
$TFP$	0.6718	0.7667	0.9011	0.9557
$R$	0.1397	0.3623	0.5307	0.6429
$\theta$	0.9982	1.1010	1.2088	1.2894
Finding rate	0.2032	0.1601	0.1390	0.1281
Seperation rate	0.0343	0.0644	0.1188	0.1312
Output	2.3426	2.3812	2.5995	2.6363

*Notes:* This table consists of the parameters and of the stochastic steady-state values used for the baseline calibration of our model described in Section 3, and for the simulation presented in Figure 1.

Table 2: Model Calibration and Stochastic Steady-State Values - HM-Style Calibration.

	Baseline - France (1)	No firing costs (2)	No notice (3)	No firing restrictions (4)
Parameter values				
$p$	0.4500	0.4500	0.4500	0.4500
$c$	2.3350	2.3350	2.3350	2.3350
$\lambda$	0.1814	0.1814	0.1814	0.1814
$\beta$	0.0520	0.0520	0.0520	0.0520
$\phi$	0.7500	0.7500	62.7500	62.7500
$F$	0.6185	-	0.6185	-
$z$	1.4269	1.4269	1.4269	1.4269
$r$	0.0100	0.0100	0.0100	0.0100
$\delta$	0.0200	0.0200	0.0200	0.0200
$\alpha$	0.3300	0.3300	0.3300	0.3300
$G(x)$		$G(x) = 1 - \left(\frac{1}{x}\right)^{1.61}$		
$q(\theta)$		$\frac{1}{(1+\theta^{0.4983})^{0.4984}}$		
Model stochastic steady state				
$u$	0.1477	0.1989	0.2466	0.2695
$n$	0.0401	0.0530	0.0010	0.0011
$e$	0.8122	0.7481	0.7524	0.7294
$\bar{x}$	1.9190	2.1212	2.2986	2.4239
$TFP$	0.6729	0.7098	0.7830	0.8112
$R$	0.1338	0.2225	0.2951	0.3503
$\theta$	0.9946	1.2301	1.4665	1.6605
Finding rate	0.2030	0.1994	0.1971	0.1947
Separation rate	0.0353	0.0496	0.0867	0.0966
Output	2.3387	2.3813	2.5948	2.6532

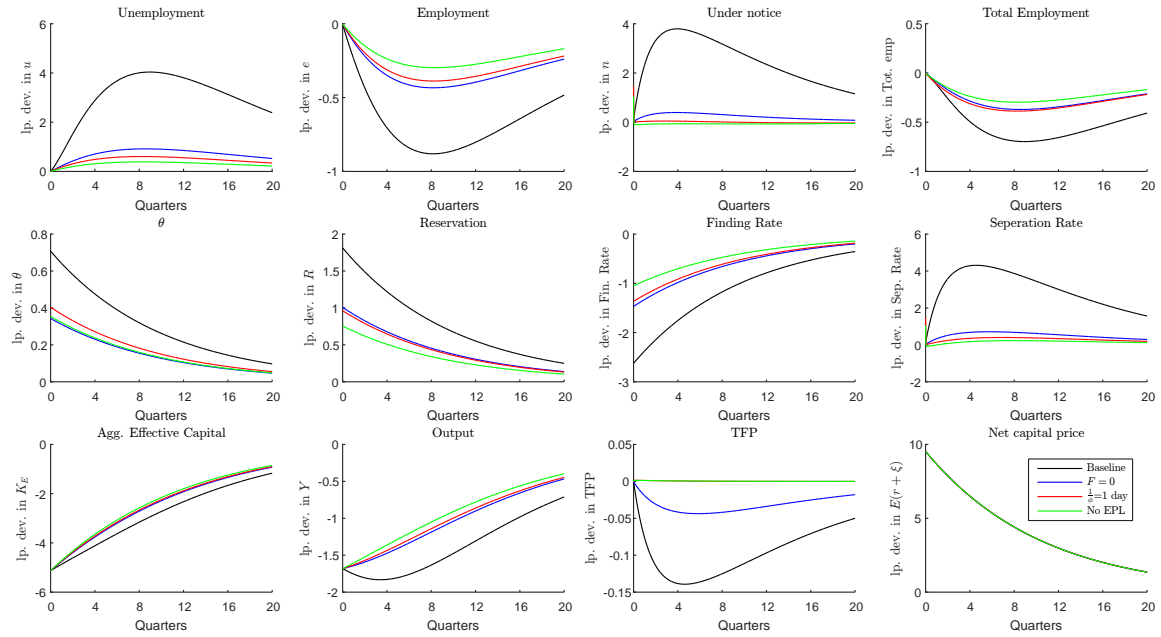
*Notes:* This table consists of the parameters and of the stochastic steady-state values used for the HM-style calibration of our model described in Section 3, and for the simulation presented in Figure 2.

Table 3: EPL: Components and Weights.

EPL index	Weights	OECD main series	Weights	OECD basic series
EPR v1 - regular contracts	33.3%	Procedural inconvenience	50.0%	Notification procedures
			50.0%	Delay involved before notice can start
	33.3%	Notice and severance pay for no-fault individual dismissal	14.3%	Length of the notice period at 9 months tenure
			14.3%	Length of the notice period at 4 years tenure
			14.3%	Length of the notice period at 20 years tenure
			19.0%	Severance pay at 9 months tenure
			19.0%	Severance pay at 4 years tenure
			19.0%	Severance pay at 20 years tenure
			33.3%	Difficulty of dismissal
	25.0%	Length of trial period		
	25.0%	Compensation following unfair dismissal		
	25.0%	Possibility of reinstatement following unfair dismissal		

Notes: The weights and the basic series are those used by the OECD and retrieved from <http://www.oecd.org/els/emp/oecdindicatorsofemploymentprotection.htm>.

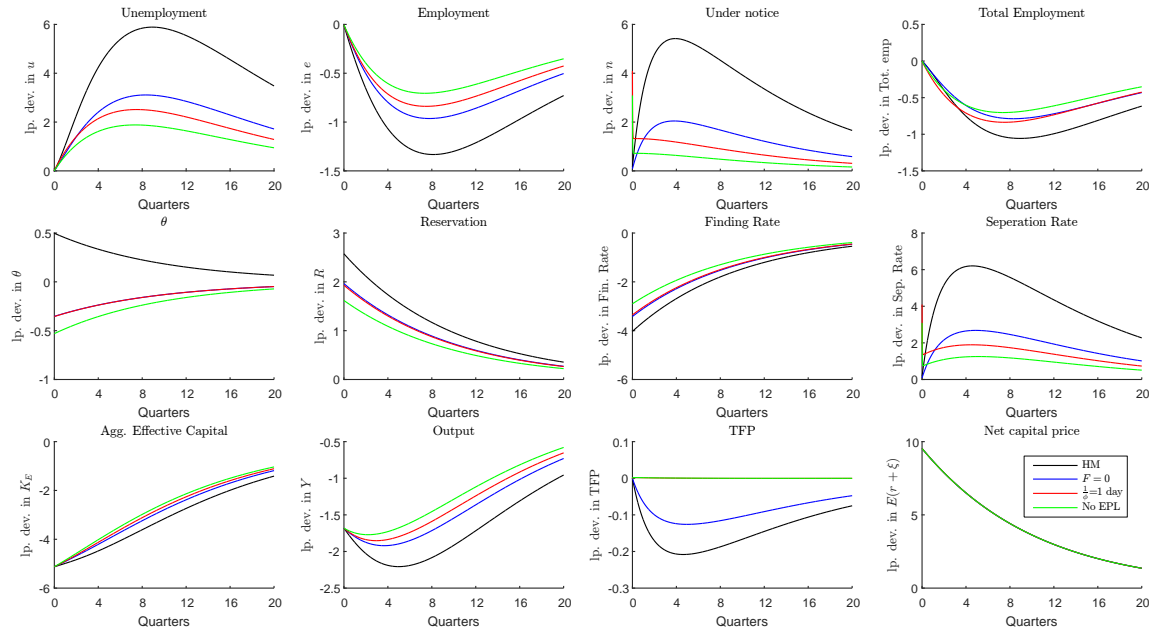
Figure 1: Theoretical Impulse Responses For Baseline Calibration and Counter-Factual Parameterizations.



*Notes:* Theoretical impulse response functions for each variable to a realization of the high risk premium state. Impulse responses for our baseline France calibration and the counter-factual ones given in columns 1 through 4 of Table 1 are presented in black, blue, red, and green correspondingly. Time horizon is in quarters and the vertical axis' units are the log-point changes from steady-state level of each variable in response to the shock.

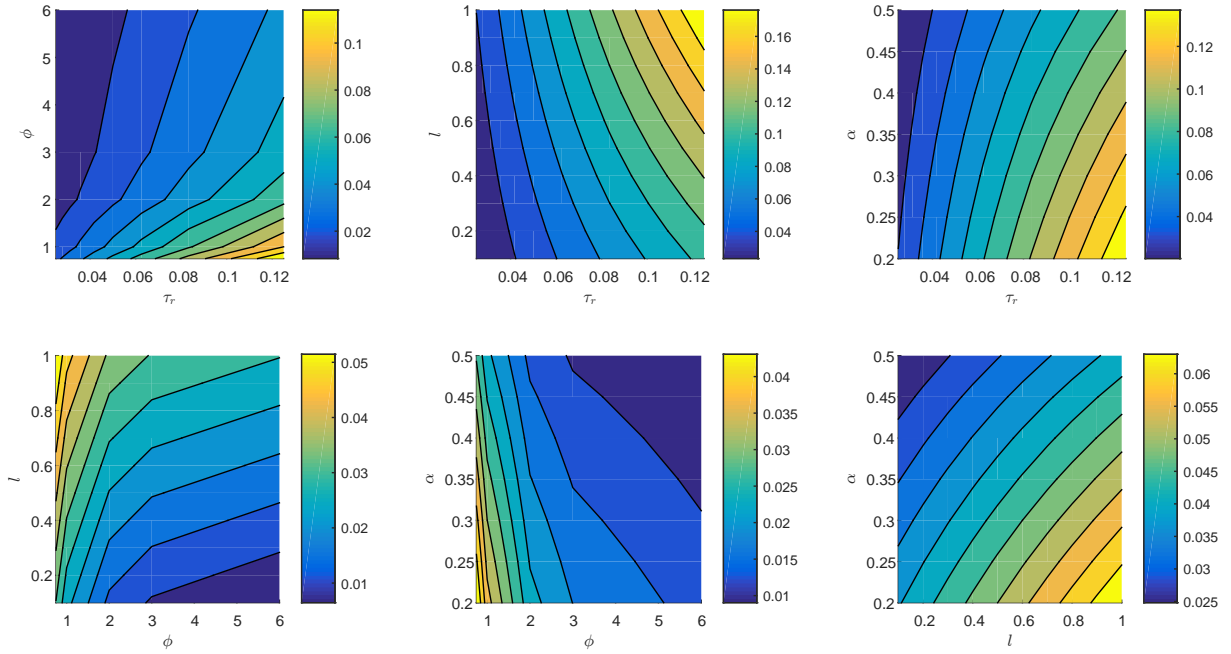


Figure 2: Theoretical Impulse Responses For Baseline Calibration and Counter-Factual Parameterizations - HM-style Calibration.



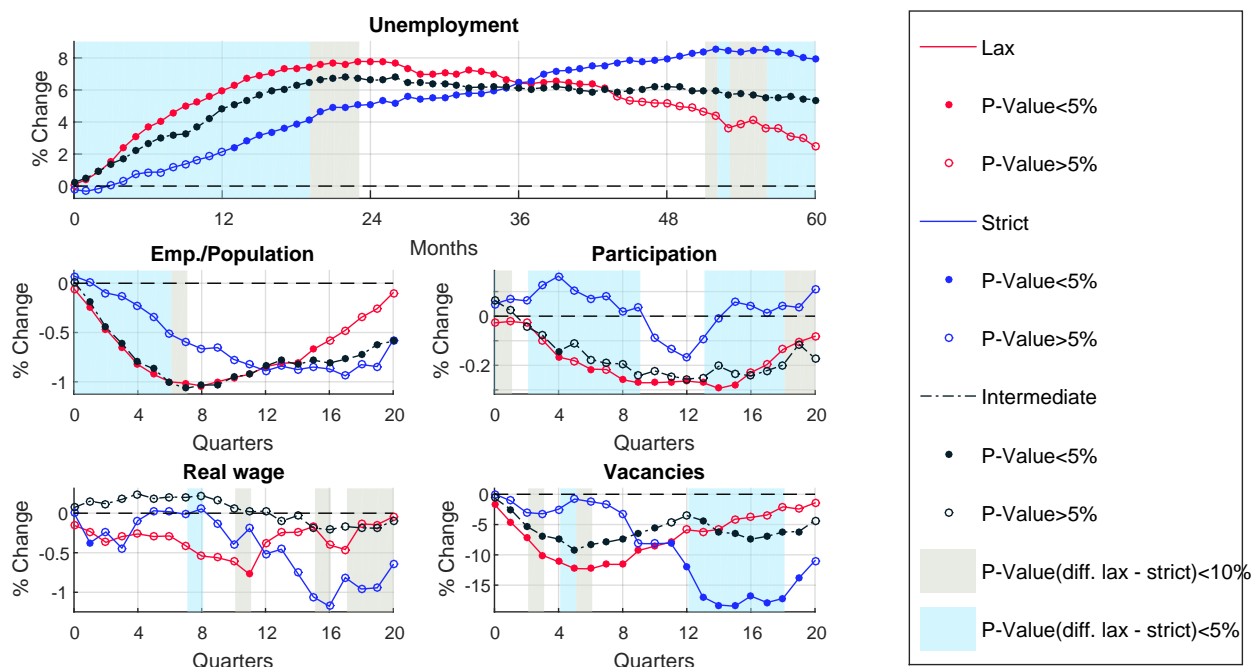
*Notes:* Theoretical impulse response functions for each variable to a realization of the high risk premium state. Impulse responses for our baseline France calibration and the counter-factual ones given in columns 1 through 4 of Table 2 are presented in black, blue, red, and green correspondingly. Time horizon is in quarters and the vertical axis' units are the log-point changes from steady-state level of each variable in response to the shock.

Figure 3: Sensitivity Analysis for the Value  $\psi$ .



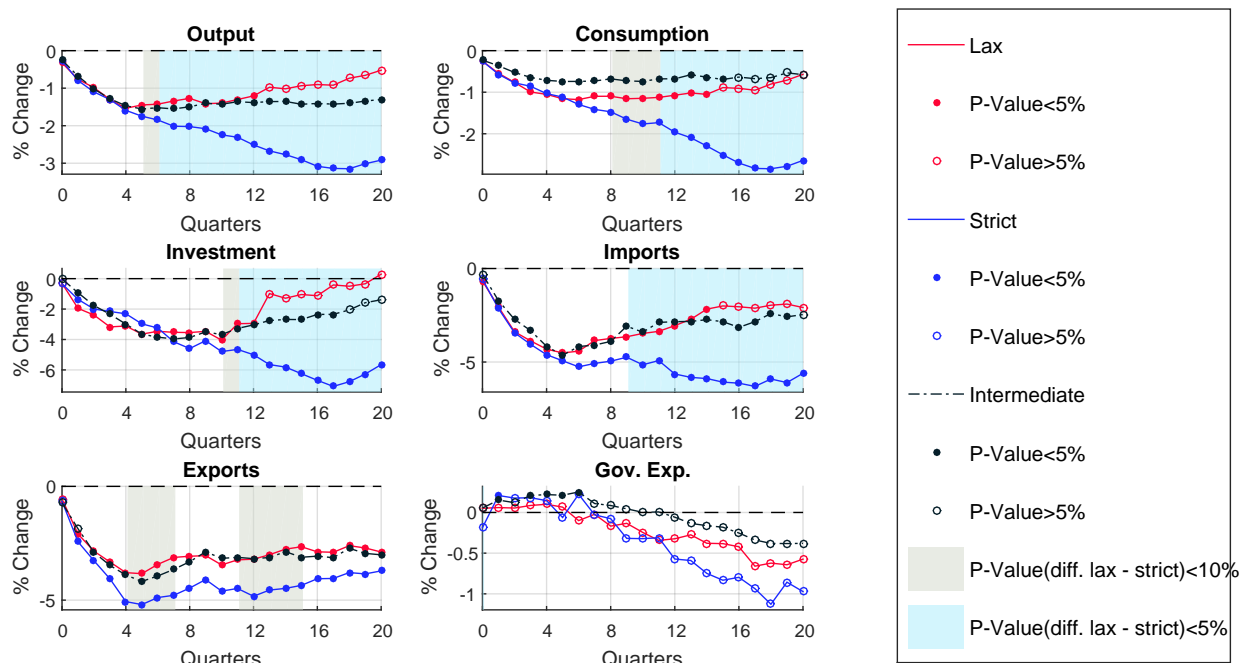
*Notes:* Each panel gives the possible values of  $\psi$  given the the parameter values that correspond to our baseline calibration in Table 1. Namely, a capital share of  $\alpha = 0.33$ ,  $\phi = 0.75$ , a termination rate of  $\tau_r = 0.0356$ , firing costs ratio to average quarterly production value of  $l = 0.33$ , and  $x_{\min} = 0$ .

Figure 4: Impulse Responses to a One Standard Deviation Credit Supply Shock Under Different EPL Regimes: Labor Market Variables.



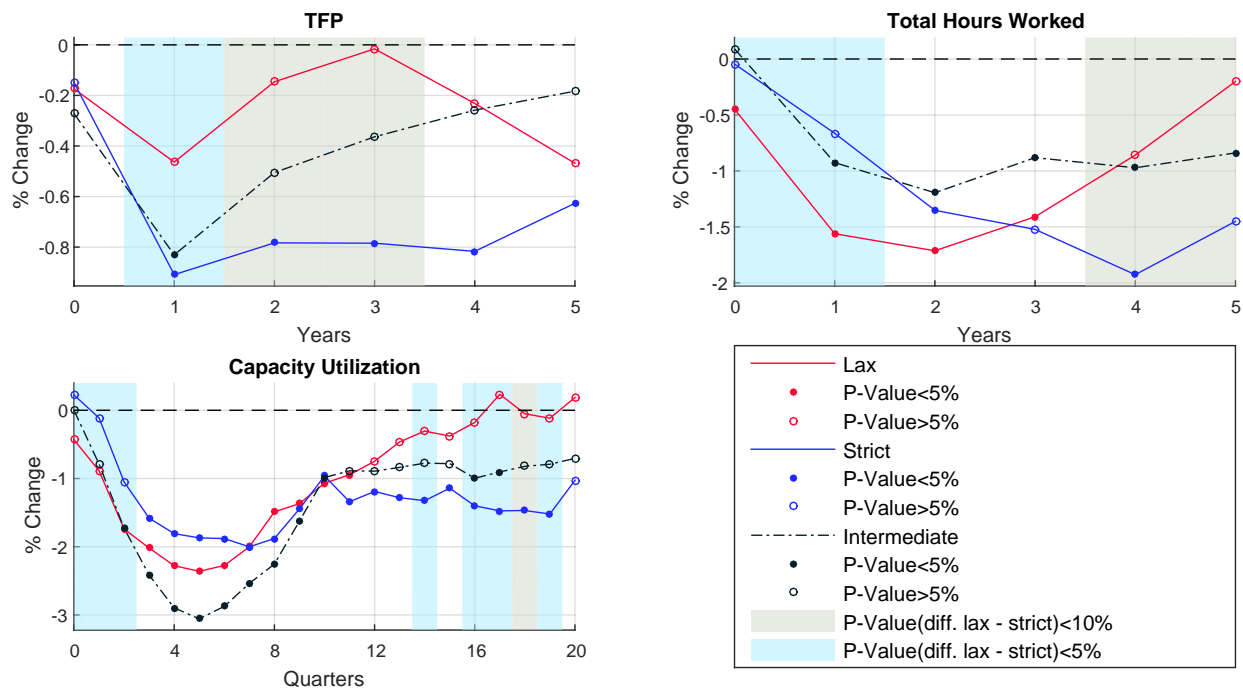
Notes: Impulse response functions for each outcome measure estimated using the state-dependent model described in Equation (32). The IRF for strict EPL regime is presented in blue, the IRF for the lax EPL regime in red and the intermediate regime in black. Full data points represent horizons at which the point estimate for the IRF is statistically significantly different than zero ( $p\text{-value} \leq 0.05$ ). Shaded areas indicate that the difference in response between the strict and lax groups is significantly different from zero ( $p\text{-value} \leq 0.05$  in light-blue shading and  $p\text{-value} \leq 0.1$  in gray). Inference is based on [Driscoll and Kraay \(1998\)](#) standard errors.

Figure 5: Impulse Responses to a One Standard Deviation Credit Supply Shock Under Different EPL Regimes: National Accounts.



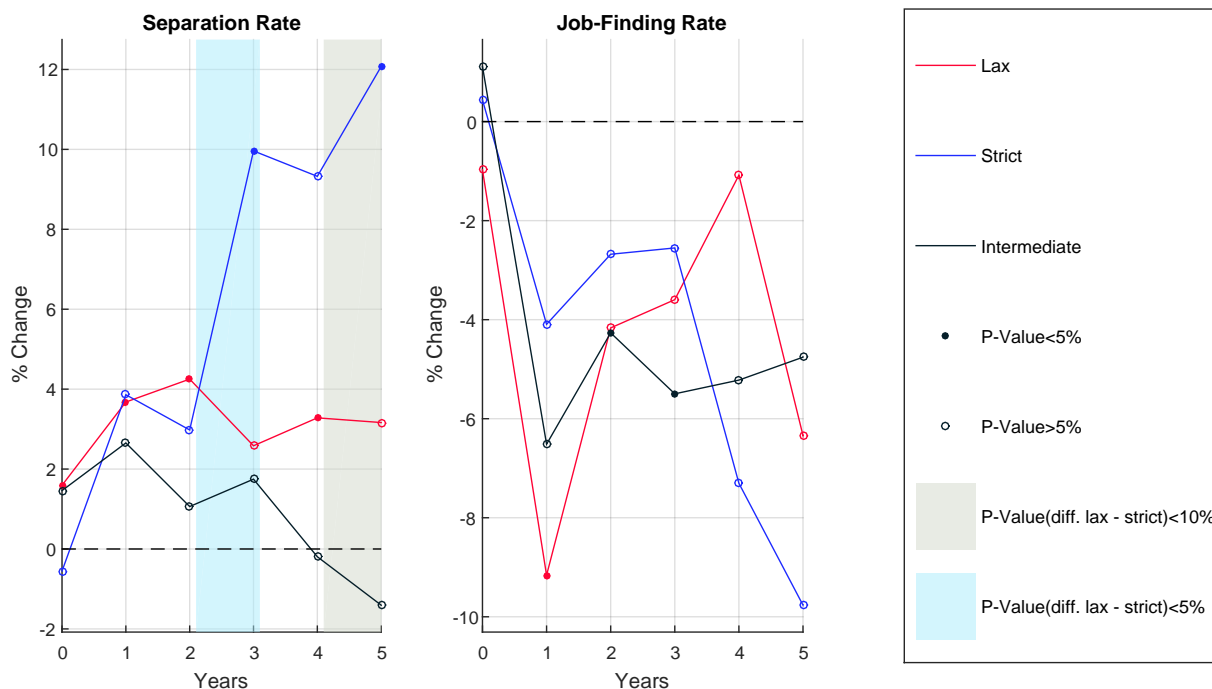
*Notes:* Impulse response functions for each outcome measure estimated using the state-dependent model described in Equation (32). The IRF for strict EPL regime is presented in blue, the IRF for the lax EPL regime in red and the intermediate regime in black. Full data points represent horizons at which the point estimate for the IRF is statistically significantly different than zero ( $p\text{-value} \leq 0.05$ ). Shaded areas indicate that the difference in response between the strict and lax groups is significantly different from zero ( $p\text{-value} \leq 0.05$  in light-blue shading and  $p\text{-value} \leq 0.1$  in gray). Inference is based on [Driscoll and Kraay \(1998\)](#) standard errors.

Figure 6: Impulse Responses to a One Standard Deviation Credit Supply Shock Under Different EPL regimes: TFP, Hours Worked, and Utilization.



Notes: Impulse response functions for each outcome measure estimated using the state-dependent model described in Equation (32). The IRF for strict EPL regime is presented in blue, the IRF for the lax EPL regime in red and the intermediate regime in black. Full data points represent horizons at which the point estimate for the IRF is statistically significantly different than zero ( $p\text{-value} \leq 0.05$ ). Shaded areas indicate that the difference in response between the strict and lax groups is significantly different from zero ( $p\text{-value} \leq 0.05$  in light-blue shading and  $p\text{-value} \leq 0.1$  in gray). Inference is based on [Driscoll and Kraay \(1998\)](#) standard errors.

Figure 7: Impulse Responses to a One Standard Deviation Credit Supply Shock Under Different EPL regimes: Separation Rate and Job-Finding Rate.



Notes: Impulse response functions for each rate estimated using the state-dependent model described in Equation (32) are presented in the second. The IRF for strict EPL regime is presented in blue, the IRF for the lax EPL regime in red and the intermediate regime in black. Full data points represent horizons at which the point estimate for the IRF is statistically significantly different than zero ( $p\text{-value} \leq 0.05$ ). Shaded areas indicate that the difference in response between the strict and lax groups is significantly different from zero ( $p\text{-value} \leq 0.05$  in light-blue shading and  $p\text{-value} \leq 0.1$  in gray). Inference is based on [Driscoll and Kraay \(1998\)](#) standard errors.